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The Distribution of Blue-Violet Light in the Solar Corona on August 30, 1905, as Derived from Photographs Taken at Kalaa-es-Senam, Tunisia

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VIII. *The Distribution of Blue-Violet Light in the Solar Corona on August* 30, 1905, *as derived from Photographs taken at Kalaa-es-Senam, Tunisia.*

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Communicated by the Joint Permanent Eclipse Committee.

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[Plate 1.]

Contents.

Table I., § 4 (*c*), and §§ 7 and 8 contain the results derived from the measurements made on the photographs, viz., equal-intensity curves of the corona and a formula (D) expressing the intensity of the corona as a function of the mean distance from the solar limb of an equal-intensity curve.

§ 1. *The Apparatus.*

The object of my expedition to Kalaa-es-Senam, Tunisia, was to obtain a series of photographs from which might be determined the distribution of light in the corona.

In designing my apparatus, I was led by two considerations: (1) the photographs had to be taken automatically, as I had to work without assistance, (2) standardising of the photographs was to be avoided. All the photographs were therefore taken on the two halves of a whole plate placed end to end and developed in the same tray during the same time. The automatic apparatus gives 10 exposures, and it is governed electrically by a pendulum clock. I employed two cameras, one with a

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Cooke triple achromatic lens of $3\frac{1}{2}$ inches aperture and 58·5 inches focal length, which belongs to the Glasgow spectrograph, the other with a Ross portrait lens of 2 inches aperture and 12 inches focal length. The pictures obtained with the larger camera are so much superior to the small size ones of the portrait lens that I have not made use of the latter in this paper. The cameras were fed by a cœlostat of 8 inches aperture, which had been kindly lent to me by the Royal Dublin Society. In front

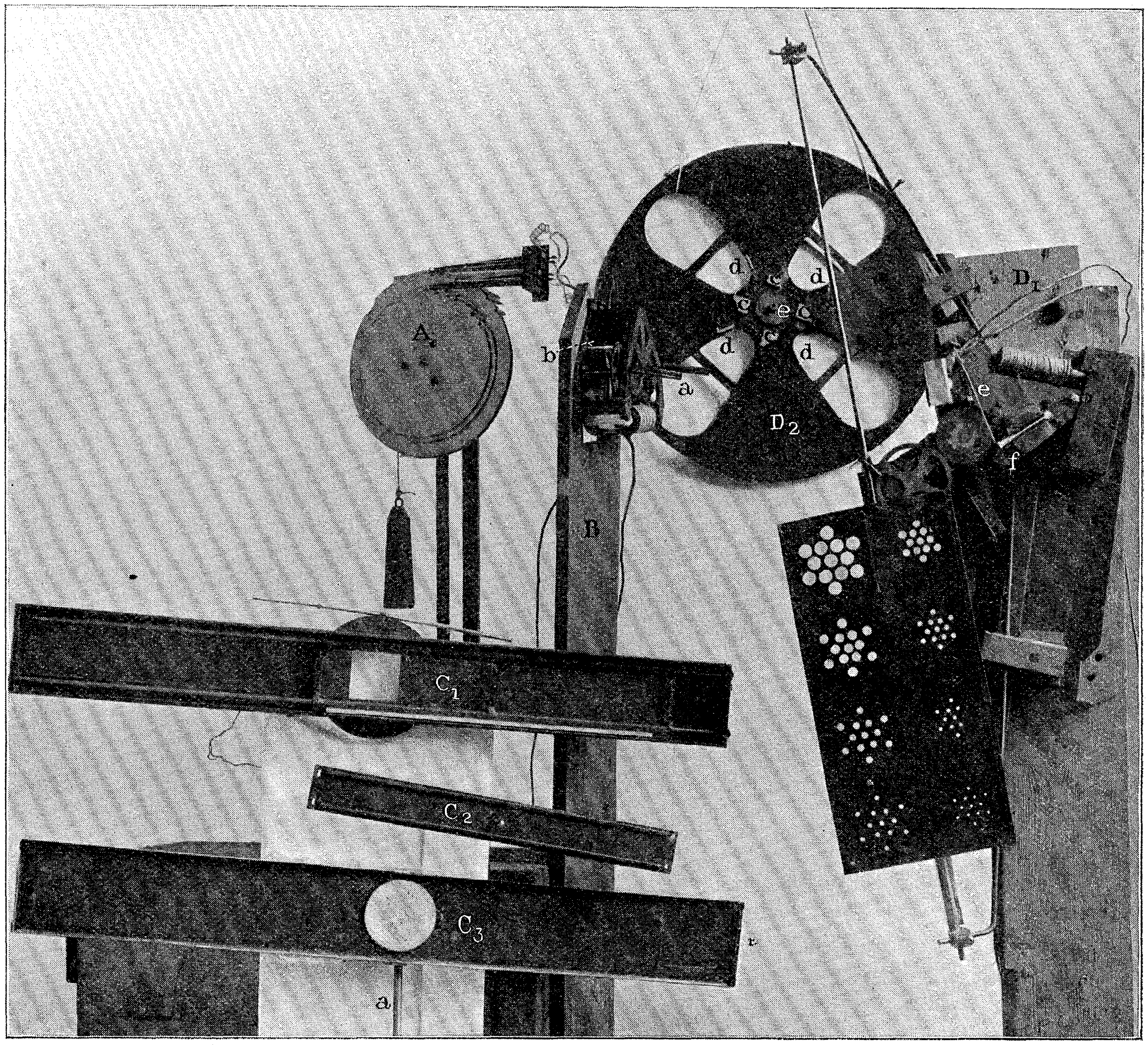

Fig. 1.

of the two object-glasses, and about an inch from them, a rotating shutter was mounted which served both cameras. The rotating shutter has four oblong apertures, 90 degrees apart (its back view is shown at D_2, fig. 1); it is rotated by clockwork driven by a spring, and its motion is governed by the armature of an electro-magnet (f). When the armature is attracted, the shutter rotates through about 45 degrees until it presses against one of the four stops d and brings an opening opposite the object-

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glasses, and when the armature is released the shutter turns again 45 degrees, as far as one of the stops *c*, and shuts off the light. The contacts are made by a pendulum clock, and they are so devised that make or break can occur only when the pendulum is at or near its position of rest.

I arranged for five exposures each of 1 second duration, and five exposures lasting respectively 3, 9, 20, 46 and 89 seconds. Their actual durations are 0·84, 0·80, 0·78, 0·80, 0·85, 2·82, 9·02, 20·84, 45·91 and 89·04 seconds, as determined automatically on the chronograph at the Observatory after my return from the eclipse expedition. I have deducted 0·02 second from the figures recorded on the chronograph to allow for the peculiar motion of the shutter. At the first four exposures of 1 second, different screens, each with 13 holes, are in front of the object-glass. The diameters of the openings are respectively 0·210, 0·296, 0·410, 0·595 inch. At the first exposure the screen leaves 1/21·4 of the object-glass free, at the second 1/10·8, at the third 1/5·6, and at the fourth 1/2·7. These screens are geared to the clockwork which rotates the shutter and fall out of gear after the fourth exposure. The illustration shows them out of gear.

The plate-holder (C) of the Cooke camera is 17×3 inches; it slides lengthways inside a metal box 32×4 inches. It is moved by rack and pinion, the rack being attached to the plate-holder, and the bearings of the axle of the pinion to the cover (C_3) of the box. Spring-driven clockwork (B) communicates its motion by means of a shaft (*a*) to the pinion. The clockwork is governed (at *b*) by the armature of an electro-magnet (the armature and the revolving stop with its axle appear white in fig. 1). When the armature is attracted, the plate-holder moves 1 inch onwards, and when it is released it moves another inch. The necessary contacts are made by the pendulum clock. I arranged the contacts in such a way that for the first four exposures the plate moves one step onwards, for all the others two steps, and when the plate has been pushed along, 2 seconds are allowed for the camera to settle before the next exposure is made. Of the 206 seconds for which I made provision, 173 seconds are occupied by the exposures, 15 seconds are taken up by changing of plates, and 18 seconds are lost.

The pendulum clock is shown at A. It is provided with four circular steel-sheet discs, into which notches are cut. The axle which carries the discs has a period of 240 seconds, *i.e.*, about half a minute more than totality lasted. Two of the discs (A, B) are represented in the diagram, fig. 2, which also shows the contact levers. The diagram gives the position immediately before making of contact. At the next second *a* will fall on *b*, making contact at *c*, and after another second *b* will fall away from *a*, thus breaking the contact. The duration of contacts depends slightly on the position of the notches, as will be seen from the figures given above for a second's contact, which show a range of 0·07 second.

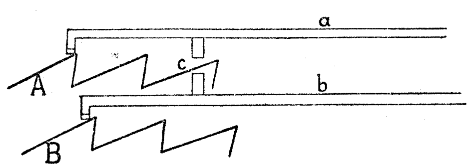

Fig. 2.

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It can be shown from the observed durations that the clock was off-beat at the Observatory, and probably it was so, too, at the eclipse, and possibly in the opposite way. This would not affect the relative duration of the first five contacts, as they all lie between an uneven and even second, but their errors would appear relative to the long exposures.

The sequence of events governed by the clock takes place at the moments of time shown in the following table, where the numerals denote the seconds elapsed from second 0, when the pendulum is started :—

Contacts for exposure.		Contacts for change of plate.		
Make.	Break.	Make.	Break.	
1	2			1. Exposure of 0·84 second, first screen, $r = 77$.
		3	—	Plate moves 1 inch.
5	6			2. Exposure of 0·80 second, second screen, $r = 55$.
		—	7	Plate moves 1 inch.
9	10			3. Exposure of 0·78 second, third screen, $r = 39$.
		11	—	Plate moves 1 inch.
13	14			4. Exposure of 0·80 second, fourth screen, $r = 27$.
		—	15	Plate moves 1 inch.
17	18			5. Exposure of 0·85 second, full aperture, $r = 16{\cdot}7$.
		19	20	Plate moves 2 inches.
22	31			6. Exposure of 9·02 seconds, full aperture.
		32	33	Plate moves 2 inches.
35	38			7. Exposure of 2·82 seconds, full aperture.
		39	40	Plate moves 2 inches.
42	131			8. Exposure of 89·04 seconds, full aperture.
		132	133	Plate moves 2 inches.
135	156			9. Exposure of 20·84 seconds, full aperture.
		157	159	Plate moves 2 inches.
161	207			10. Exposure of 45·91 seconds, full aperture.

r designates the ratio of the focal-length and the diameter of a lens, which has the same area as the lens reduced by the screen.

There are several points in the design of the apparatus which have proved unsatisfactory. The shutter must have a smaller moment of inertia, and its motion should be recorded on a chronograph; the plate-holder ought to run on wheels instead of sliding on a rod. The mutual distances of the pictures ought to be, say, four solar diameters, and, especially, the side of the square opening in front of the plate must be twice as great as the distance between the pictures [see § 5 (*f*), (*g*), (*h*)]. One of the pictures (not the last) must be 8 diameters from its neighbours [see § 4 (*c*)]. The screen for cutting down the aperture of the lens ought not to contain a series of small openings, but have a central opening and an annular opening, whose diameter is about two-thirds of that of the lens (see Appendix I, p. 332).

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§ 2. *The Photographs* (Plate 1).

The pendulum was started about a second after Mr. H. Mavor, who watched the contact, gave the signal that totality had begun. Before the last exposure was finished sunlight appeared, but I shut it off by stepping in front of the object-glass, and it was about 3 seconds before the shutter automatically closed. That is to say, totality lasted about $1+207-3 = 205$ seconds, as compared with the calculated time of 210 seconds. Accordingly, the tenth exposure lasted about 43 seconds.

The plates (two halves of a whole plate) were developed together in the same tray by a strong developer (Imperial standard) for 7 minutes. I developed at the open window at star-light, keeping the plates covered most of the time. The photographs show a great deal of contrast, and this has proved an advantage in measuring them. The background of the long-exposed negatives is dense, due to the brightness of the sky. This diffused light, whose intensity I had underrated in the design of the apparatus, produced an impression even for the shortest exposures, darkening a square on the plate equal in size to the opening in the plate-holder. I find, from measurements, that the intensity of the diffused light equals that of the corona at a distance of 1·1 solar diameter from the sun's limb, *i.e.* 0·6 in unit of the intensity of the corona at a point 1 solar diameter distant from the limb as found from the formula § 7.

In the preliminary report I have said that the plate-holder failed to move in the designed manner, due to some parts of the apparatus having been damaged in transit. Owing to this accident, there is a multiplication of images in the sixth and seventh pictures, and not only the first five photographs are an inch (2 solar diameters) apart, but also the next two, which were meant to be at twice that distance. In consequence, the successive exposures to the diffused light overlap on the plate, with the effect that on one half of each of Photographs I. to VII. there is the same duration of exposure to the diffused light as on the adjoining half of the neighbouring photograph. This has enabled me to separate the intensity of the corona from that of the sky. There is no overlapping on Photographs VIII. and IX., and though they did not, on that account, furnish data for the intensity formula, they supplied a series of equal-intensity curves of the corona, which are required for the reduction of the other pictures. Fig. 3 explains the conditions.

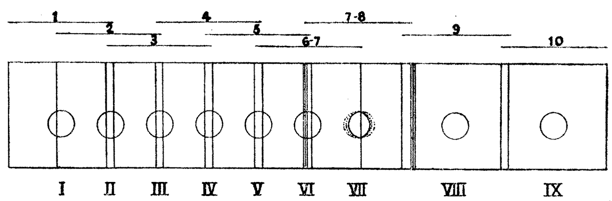

Fig. 3.

The photographs are numbered I., II., &c., and the exposures (see § 1), 1, 2, &c. The lines at the top give the extent of the area illuminated at each exposure by the

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diffused light. The left and right halves of a photograph will be designated by *a* and *b*. Photographs I. to VII. occupy the first half-plate, and VIII. and IX. half of the second half-plate.

Photographs VI. and VII. are, from an ideal point of view, marred by defects—No. VI. by some instantaneous pictures of the protuberances which appear on the lunar disc, and No. VII. by two short exposure pictures which are eccentrically superposed on it. These defects are of no consequence (see § 6). Owing to the failure of the automatic apparatus, the exposures of Nos. VI. and VII. are uncertain to about a second, but the sum of the two exposures, which equals the sum of the sixth, seventh and eighth exposures, is accurately known.

To get the negatives, from which the Plate was prepared, I first made enlarged positives, copying those pictures together whose background have the same density on the original. I have attempted to make the coronas of VI*a*. and VI*b*. extend equally far, and in the attempt the coronas of pictures V*a*. and VIII. have come out too small. I should say that on the original negative the corona of VIII. covers the whole breadth of the plate, which towards the top is three-quarters of a solar diameter broader than shown in the reproduction. The enlargement is 1·7.

§ 3. *The Measurements.*

The observations made on the photographs, and utilised in this paper, consist in the selection of points on the several corona pictures at which the photographic film shows the same degree of blackness, and in the measurement of their distance from the lunar disc. The measurements were actually made on positives, and not one but twenty-four points of an equal-blackness curve, 15 degrees apart, were measured. The positives are contact prints on slow plates obtained at a distance of 10 feet from a gas jet. The twelve sets which I prepared belong to different exposures, and were developed for contrast. I copied the negatives I. to V*a*., V*b*. and VI*a*., VI*b*. and VII. separately on account of the differences in density of the background. Sixteen sets of measurements of equal-density curves were made on these twelve sets of positives. The measurements were easy to make, and proved to be consistent. The positives show a perfectly transparent ring round the lunar disc, the diameter of which depends on the exposure and development. Seen against black paper, this ring furnishes a well-defined outline to set upon. Some of the curves at great distance from the sun, where the intensity changes little with the distance, were measured on enlargements (10 diameters) on bromide paper, in which the contrasts are much increased.

I further made twenty copies of each of Photographs VIII. and IX. at all kinds of exposures. These negatives are very dense, and, apparently, evenly dense up to about half a diameter from the sun's limb, and show no detail to the eye when inspected against a strong light. On the other hand, the positives contain detail as near as 0·12 diameter, and as distinctly as if they were replicas of the first six

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negatives. The measurements furnished 79 curves of equal density, which belong to mean distances between 0·12 and 1 solar diameter. As each of the two photographs has a background of equal density all round, these curves will be employed in reducing measured distances on other photographs as described in § 4 (*c*) and § 4 (*d*).

The apparatus which I employed in measuring the photographs consists of a low-power microscope mounted on a slide whose position can be read by vernier to 0·001 of an inch. The slide is mounted on a circular plate which turns in a ring, so that measurements can be made at any position-angle.

The measurements were made at position-angles 0 degree, 15 degrees, &c. The position-angles refer to the north pole of the sun. I obtained the zero of the position-angles from the calculated position-angles referred to the north pole of the sun and the positions of the second and third contacts on the first and last photographs. The position-angles of the contacts referred to the celestial pole were found from the data given in the 'Nautical Almanac' (124 degrees and 287 degrees), and the position-angle of the north pole of the sun is 20·7 degrees.

On the photographs published with this paper the line joining the centres of the lunar discs has a position-angle of 239 degrees, 59 degrees being to the left and 149 degrees at the top.

I observed the following rule in measuring:—After clamping the microscope at a certain position-angle, I set the wire successively on the moon's limb, then on a point of the corona where the blackness had a certain density and, without turning the microscope, made similar measurements 180 degrees from the first position. Keeping the degree of blackness in my mind, I repeated the operation on the other photographs, and then for the other position-angles.

§ 4. *Reductions.*

The object of the reductions is to find (1) the mean distance from the solar limb of each equal-density curve, and (2) the position of each equal-intensity curve with reference to its mean circular-intensity curve. The steps are as follow:—

(*a*) There is a slight difference amounting to a few thousandths of an inch between the diameters of the moon as obtained from negatives and from positives. On the negatives the lunar diameter is 0·565 of an inch, and I reduced all the measured distances to this diameter by correcting them by half the difference between this figure and the diameter appertaining to each measured distance.

(*b*) *Reduction of the Distances from Lunar Limb to Solar Limb.*—M (fig. 4) is the centre of the moon, A, B and S are respectively the centres of the sun at second and third contacts and t seconds after the second contact. The duration of totality is about 205 seconds. The diameter of the moon is $D = 0·565$ of an inch, and that of the sun is $d = 0·540$ of an inch. The angle between the second and third contacts, 156·7 degrees, is given by the first and last photographs; it equals the angle

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subtended by BA at M. The position angle, counted from the north pole of the sun, of the second contact is 104 degrees. CMS is designated by α.

A measured distance m, at position-angle P, is reduced to distance h from the sun's limb by the following formula, small quantities being neglected,

$$\tan \alpha = \left(1-\frac{t}{103}\right) \tan 78^\circ{\cdot}3, \quad h = m+0{\cdot}0125 \left[1-\frac{\cos 78^\circ{\cdot}3}{\cos \alpha} \cos (\mathrm{P}+\alpha-104-78)\right].$$

The maximum of $h-m$ is 0·025 inch. The correction of the position-angles is inappreciable for our purpose.

(*c*) *Curves of Equal Intensity of the Corona.*—I define the mean distance of an equal-intensity curve of the corona as the mean of the distances of twenty-four points of the curve, 15 degrees apart. Equal-blackness curves coincide with equal-intensity curves on Photographs VIII. and IX., and also on Photograph I. The measured distances were first corrected for corrections (a) and (b), and then each twenty-four

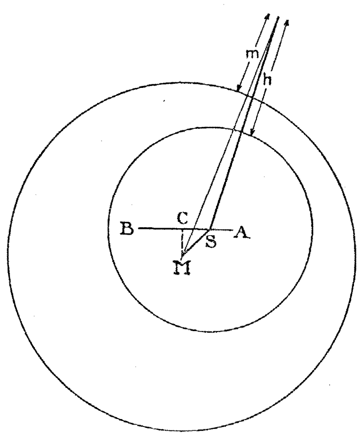

Fig. 4.

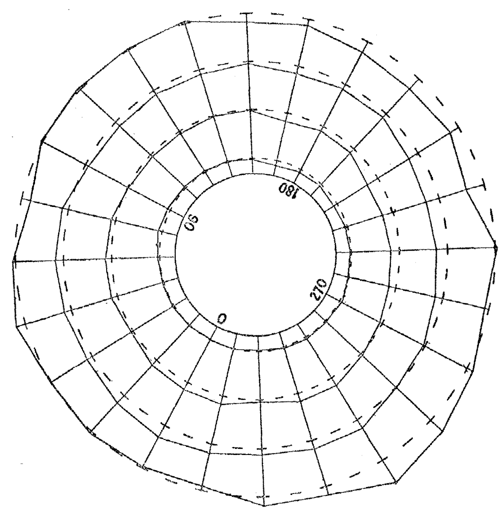

Fig. 5.

distances belonging to an equal-intensity (or blackness) curve were combined to a mean; the differences, δh, (mean minus reduced distance) define the equal-intensity (or blackness) curve with reference to the circular mean curve. Finally curves were interpolated from the observed 95 curves at regular intervals of the mean distance. An extract of the results is contained in Table I. (p. 337), and graphs of some of the curves are shown in fig. 5.

(*d*) *Reduction of the Distances of Portions of an Equal-blackness Curve to the Mean Distance of that Curve.*—Though equal-blackness curves were measured on all pictures at all position angles, only portions of these curves can be used together, because the equal-blackness curves do not everywhere coincide with the equal-intensity curves of the corona. In next section it will be shown (1) that in the case of Photographs V., VI. and VII., owing to luminosity of the sky, the left and right

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halves of an equal-blackness curve coincide each with an equal-intensity curve of the corona, though not with the same curve of the corona, and that on the two halves of Photographs I. to IV. an equal-blackness curve differs inappreciably from an equal-intensity curve; (2) that on account of the overlapping of the coronas belonging to neighbouring pictures the intensity and blackness curves do not coincide at certain position-angles. Therefore, if the mean distance of each equal-blackness curve be derived separately for each half of Photographs V., VI. and VII., and all measurements be excluded which belong to position-angles where there is appreciable overlapping of coronas, the mean distance will also be the mean distance of an equal-intensity curve. Each measurement belonging to a position-angle p, and reduced in accordance with (a) and (b), plus the correction δh derived above (c), gives a mean distance of the equal-blackness curve, and there are as many values of this mean distance as there are measurements. Their average value h is the final value, and its error can be determined from the differences from the mean. Table III. contains h, its error and the number of measurements p which contribute to the mean value. $p = 24$ indicates that all the points of the curve, position-angles 0 to 345, were used. For $p = 21$, the points at position-angles 225, 240, 255 degrees are excluded and for $p = 19$ those at 210 and 270 degrees are omitted in addition. For Photographs Vb. to VII. the points omitted lie symmetrically round position-angles 60 degrees and 240 degrees. The quantities are given in unit of 10^{-3} solar diameter, those derived in inches being multiplied by 1·852 (diameter of moon on photograph 0·565 of an inch, diameters of moon and sun 994·5 and 950·7 seconds). I designate by "corresponding distances" the distances from the sun's limb of two points on two different pictures of the corona at which there is equal blackness. In Table III. the mean corresponding distances stand on the same line. I shall show in the next section that at these tabulated distances the ratio of the intensities of the corona is a constant for each two photographs.

§ 5. *Correlative Distances on Corona.* [*Definition see under* (d).]

(a) I employ the following notation. I or S is an intensity of light acting on a photographic plate, and they are the quantities of light falling on unit area of the plate, which is the area cut out in the focal plane of the camera by unit of spherical angle at the centre of the object-glass. i or s is an intensity of a luminous object, *i.e.*, a quantity of light falling from unit area of object (area cut out by unit of spherical angle) on unit area of the object-glass, which unit area equals that for the plate. a designates the exposed area of the object-glass and t the time during which the plate is illuminated. Then $\mathrm{I} = as$ and $\mathrm{S} = as$.

For the pattern of screens by which I reduced the aperture of the lens the loss of light due to the object-glass will be about proportional to the aperture, and it need not be taken into account, but the effects of diffraction require special investigation (see Appendix I.).

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(*b*) *Intensity and Time for Equal Blackness.*—Experiments have proved that for the same photographic plate and development the intensities of light and the durations of exposure necessary to produce the same blackness on the film bear a certain relation to each other. According to MICHALKE this relation is independent of the degree of blackness. I have represented MICHALKE's observations by the formula $tI^{\alpha} = \text{constant}$, where $\alpha = 1{\cdot}08$, and further redetermined α for the plates employed by me (Imperial special rapid). I used as a source of light a disc of opal glass (2 inches in diameter), illuminated from behind by an electric lamp, and I exposed directly to the light of the disc successively different portions of the same plate at distances varying from 1 to 15 metres. A Thornton-Pickard shutter recorded the duration of exposure automatically on a chronograph. My value of α is $1{\cdot}05 \pm 0{\cdot}01$.

(*c*) The ratio of two intensities, i_m, i_n, which illuminate, through apertures a_m, a_n of a lens during times t_m, t_n, a photographic plate placed in the focus of the same camera is a constant if they produce equal blackness on the film. By (*a*) and (*b*)

$$\frac{i_m}{i_n} = \frac{a_n}{a_m}\left(\frac{t_n}{t_m}\right)^{1/\alpha} = F_{mn}.$$

F_{mn} can be calculated for the eclipse photographs. The individual exposures of Photographs VI. and VII. are uncertain to about a second (see § 6), but their sum is accurately known (100·88). I take here $t_6 = 11{\cdot}00$, and hence $t_7 = 89{\cdot}88$. The other data are given in § 1. The numerical value of α is of no importance for the first five photographs.

m, n	1, 2	1, 3	1, 4	1, 5	5, 6	6, 7.
$\log F_{mn}$	0·278	0·551	0·884	1·335	1·059	0·869.

(*d*) *Correlative Distances on the Corona.*—If the pictures of the corona had not been overlapping, and the sky been dark, an equal-blackness curve would have coincided with an equal-intensity curve of the corona, and the ratio of the intensities of the corona belonging to two such curves on Photographs *m* and *n* would equal a constant F_{mn} [see (*c*)]. I shall call "correlative distances on corona" the distances of points of the corona at which the ratio of the intensities equals F_{mn}.

(*e*) *Simultaneous and Successive Exposures.*—I make the following two assumptions :—(1) the degree of blackness on the film is independent of the order in which two or more exposures are made ; (2) if two intensities give the same blackness for certain exposures, they do so, too, when these exposures are made on an otherwise exposed film. I have checked (1), but not (2), by experiment.

Let two intensities I and S illuminate the film together during the same time *t*. By (*b*)

$$(I+S)^{\alpha}t = S^{\alpha}(t+t') = S^{\alpha}t + S^{\alpha}t' \text{ for equal blackness,}$$

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where t' can be determined from the equation. The formula expresses that (I+S) acting on the film during t gives the same blackness as S acting during t and t', t' being, of course, after (or before) t. Hence the positive sign stands for "the one exposure after the other." Each of the terms may be replaced by a term of the form $a^{\alpha}b$, which equals it in value [see (2)], and a expresses the intensity, b the time. The terms may be written in any order [see (1)].

(*f*) *Elimination of the Diffused Light of the Sky.*—Let i and I belong to the corona, s and S to the sky. On the second half of the m^{th} photograph I_m+S_m illuminates the film during t_m, and thereafter S_{m+1} during t_{m+1}; on the first half of the $(m+1)^{\text{th}}$ photograph S_m illuminates the film during t_m, and thereafter $I_{m+1}+S_{m+1}$ during t_{m+1}. Let both produce equal blackness. By (*e*)

$$(I_m+S_m)^{\alpha}t_m+S^{\alpha}_{m+1}t_{m+1} = \text{constant} = S^{\alpha}_m t_m+(I_{m+1}+S_{m+1})^{\alpha}t_{m+1}$$

$$I'^{\alpha}_m t_m+S^{\alpha}_m t_m+S^{\alpha}_{m+1}t_{m+1} = I'^{\alpha}_{m+1}t_{m+1}+S^{\alpha}_m t_m+S^{\alpha}_{m+1}t_{m+1}.$$

The last two terms disappear, and therein lies the advantage introduced for Photographs V. to VIII. by the failure of the mechanism during the eclipse;

$$I'^{\alpha}_m t_m = I'^{\alpha}_{m+1}t_{m+1}, \quad \text{where} \quad I' = I\left[\left(\frac{S}{I}+1\right)^{\alpha}-\left(\frac{S}{I}\right)^{\alpha}\right]^{1/\alpha}.$$

Substitute i and s [see (*a*)] and introduce F by (*c*); therefore

$$\frac{i'_m}{i'_{m+1}} = F_{m,\,m+1}, \quad \text{where} \quad i'_m = i_m\left[\left(\frac{s_m}{i_m}+1\right)^{\alpha}-\left(\frac{s_m}{i_m}\right)^{\alpha}\right]^{1/\alpha}.$$

Equal blackness was observed at the distances h_m and h_{m+1}, hence h_m and h_{m+1} are corresponding distances; they are, however, not correlative distances on the corona, because the ratio of i_m and i_{m+1}, the intensities of the corona at h_m and h_{m+1}, does not equal $F_{m,\,m+1}$. On the other hand, the distances $h_m+\Delta h_m$, $h_{m+1}+\Delta h_{m+1}$, at which the intensities of the corona equal i'_m and i'_{m+1}, are, by definition (*d*), correlative distances on the corona. Hence we have

$$\Delta h_m = (\log i'_m-\log i_m)\left(\frac{dh}{d\log i}\right)_m.$$

I calculate $\log i$ and the differential quotient by the formula derived in this paper, which gives i as a function of h; further, i' for $s = 0{\cdot}6$, and thence Δh. The values are: $\Delta h = 0$ for $h = 200$, $\Delta h = -3$ for $h = 600$, $\Delta h = -15$ for $h = 1000$, and $\Delta h = -35$ for $h = 1400$. The measured corresponding distances are correlative distances on the corona with an error Δh. These systematic errors are insignificant compared with the accidental errors of measurement (see Table III.) up to $h = 800$, and even for the most distant parts of the corona they do not reach these accidental errors. The correlative distances determine the intensity formula (see § 7), and in the equations the residuals appear under the form $v = \Delta h_m - F^{1/4}_{m-1,\,m}\,\Delta h_{m-1}$. I observed on

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pictures VI*b*. and VII. the corresponding distances $h_6 = 550$, $h_7 = 1000$, and $h_6 = 900$, $h_7 = 1400$; hence v equals -10 for $h = 1000$, and -17 for $h = 1400$, while the accidental errors v are about 45 and 75, that is to say, four times as great as the systematic errors. Compared with the actual residuals v left by the equations, the systematic errors are still smaller. The result, then, is this: Let there be equal blackness on the two adjoining halves of two neighbouring pictures (Nos. m and $m+1$) of the corona at distances h_m and h_{m+1}, then h_m and h_{m+1} are also correlative distances on the corona at which the ratio of the intensities equals $F_{m,\,m+1}$.

The result would have been different if the backgrounds of the two neighbouring pictures had not been overlapping. $i' = i+s$ would have been found instead of i, and s could not have been separated from i.

(*g*) The intensity of the diffused light of the sky can be disregarded on the first five photographs. To prove this, I start from the equations [see (*f*)]

$$(I_m+S_m)^{\alpha} t_m + S^{\alpha}_{m\pm1} t_{m\pm1} = \text{constant for equal blackness.}$$

The lower sign belongs to the first half of the mth photograph, and the upper sign to the second half. For the first five photographs s is small compared with i, hence by (*a*), (*b*), and (*c*)

$$a^{\alpha}{}_m (i_m+\sigma_m)^{\alpha} t_m = \text{constant}, \quad \text{where} \quad \sigma_m = s\,(1+F_{m,\,m\pm1})$$

$$\frac{i'_m}{i'_n} = F_{mn}, \qquad \text{where} \quad i'_p = i_p+\sigma_p.$$

σ_m is the intensity which produces at aperture a_m and exposure t_m the same blackness as the two superposed exposures to the diffused light. The distances belonging to the intensities i'_m and i'_n are, by definition, correlative distances on the corona. We obtain, then, in the same manner as explained in (*f*),

$$\Delta h_m = \sigma_m \left(\frac{dh}{di}\right)_m$$

and

$$v = \Delta h_m - F^{1/4}{}_{1,\,m}\,\Delta h_1 = \Delta h_m - F^{-1}{}_{1,\,m}\left(\frac{di}{dh}\right)_1 \left(\frac{dh}{di}\right)_m \Delta h_1 = (\sigma_m - \sigma_1 F^{-1}{}_{1,\,m})\left(\frac{dh}{di}\right)_m.$$

σ_m is calculated by the above formula and $s = 0{\cdot}6$, except for the first photograph and the first half of the second. The values range between 0·8 and 2·3.

A minute before the beginning of totality the cap was removed from the object-glass and during that time light must have been reflected into the camera by the shutter, which was placed about an inch from the object-glass, and illuminated the plate at the place where Photographs I. and II*a*. were taken. The blackness of the background lies between that of IV. and V., and I estimate $\sigma_1 = 10$ and $\sigma_2 = 5$.

The calculated values of v do not amount to a third of the accidental errors v of measurement (Table III.). It is, therefore, permissible to regard the corresponding

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distances measured on pictures I. to Va. as correlative distances on the corona, just as if no diffused light had been illuminating the plate.

After the experience gained at this eclipse I should again place the pictures as they appear in the diagram, *i.e.* make the opening of the plate-holder twice as long on a side as the distance between the pictures. This arrangement entails no disadvantage for the short exposures, and for the long exposures the intensity of the diffused light can be eliminated. (See § 1, last section.)

(*h*) *Overlapping of Coronas of Neighbouring Pictures.*—At a point A of the m^{th} picture the intensity of the corona is (i_m) at distance h_m, and the intensity of the light which illuminates A for a time t_m is $(\mathrm{I}_m) = a_m(i_m)$. The same point is illuminated for a time $t_{m\pm1}$, also by light of intensity $(\mathrm{I}_{m\pm1})$ belonging to a different part of the corona in the $(m\pm1)^{\text{th}}$ photograph, where the corona has an intensity $(i_{m\pm1})$ and $(\mathrm{I}_{m\pm1}) = a_{m\pm1}(i_{m\pm1})$. Point A lies on an equal-blackness curve of the m^{th} picture, and this curve coincides with an equal-intensity curve of the corona (intensity $= i_m$) at all points where there is no overlapping. By (*c*)

$$[a_m i_m]^a t_m = [a_m(i_m)]^a t_m + [a_{m\pm1}(i_{m\pm1})]^a t_{m\pm1},$$

or very nearly

$$i_m = (i_m) \qquad + (i_{m\pm1})\,\mathrm{F}_{m,\,m\pm1}.$$

The equal intensity curve (intensity $= i_m$) cuts the radial line belonging to A (distance $= h$) at A′, and AA′ $= \Delta h$ is the distance of the two curves at A. Hence

$$\Delta h = (i_{m\pm1})\,\mathrm{F}_{m\pm1}\frac{dh}{di}.$$

I measured on a diagram the distances of a point A from the solar limbs of the following and preceding pictures, calculated $i_{m\pm1}$ by the formula $i = f(h)$ and thence Δh. In deriving the mean distance of an equal-blackness curve [preceding section (*d*)] I used only those measured distances for which the average value of Δh (including $\Delta h = 0$) is less than a half of the calculated error of the average distance. The number of values is given in Table III. under heading p [see preceding section (*d*)]. The average values of Δh increase with the accidental error, but they have always the same sign, so that the systematic residuals v become very small compared with the accidental errors v. It would of course have been better if all the images had been further apart. (See § 1, last section.)

The outcome of the discussion given in this section is, that the mean corresponding distances given in Table III. are also mean correlative distances on the corona.

§ 6. *Photographs Nos. VI. and VII.*

(*a*) *Duration of Exposure.*—Owing to the failure in the driving of the plate-holder only two pictures (VI. and VII.) belong to the three exposures 9·02, 2·82, 89·04 seconds. The sum of the durations of exposure of these pictures is thus given

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(100·88). It is of some importance to know the upper limit, if not the accurate durations, of the exposure belonging to Photograph VI. On Photograph No. VII. three images are eccentrically superposed (see fig. 6). The order of magnitude of the time for which they were illuminated can be found from the degree of blackness of the background which belongs to each image. In this way I find that circle (b) is due to a short exposure of the order of a second, the faint circle (c) (dotted in fig. 6) which is faintly visible within the corona is the lunar disc during a very long exposure, and semi-circle (a) must belong to an exposure of about 10 seconds. Now (b) is exactly at the position on the plate, as determined by its distance from other pictures, at which the plate-holder was locked by the electromagnet of the propelling mechanism. The contacts made by the clock for unlocking and locking the plate-holder are between the contacts for exposures, and hence no part of the exposure which produced (c) can have contributed to (b). As (c) belongs to the longest exposure, (b) must be due to some portion of the 2·8-seconds exposure. The first part of this exposure must have contributed to picture VI., because the plate moved (see the instantaneous photographs of the protuberances on the lunar disc of picture VI. and in the coronas of VI. and VII.) while an exposure was going on. The exposure given to No. VI. is therefore $9{\cdot}02+2{\cdot}82-\tau = 11{\cdot}84-\tau$, and the combined exposure for abc of No. VII. is 100·88, less the exposure of No. VI. τ may lie between 0·1 and 1 second (see above). I should add that the lunar disc on Photograph VI. is exactly round and not blurred in the least. Apart from the instantaneous pictures and trails of prominences on the lunar disc, the picture VI. is perfect.

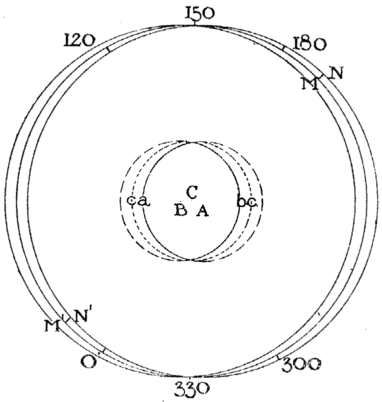

Fig. 6.

(b) *Measurement of Curve of Equal Blackness on Photograph VII.*—I shall now investigate whether the three eccentrically superposed images on this photograph can be utilised in this research. Let the curves of equal intensity of the corona be circles, and let us consider the curves at some distance from the sun. Let C and A (fig. 6) be the centres of two photographs of the sun (not moon, as it was supposed in No. 6a) and MM′ and NN′ be circles of radius r along which the corona has an intensity i. To centre C belongs the exposure t_1 and to A the exposure t_2. Along circle MM′, i has been exposed during t_1 and on this is superposed an exposure to $i-(di/dr)\,\Delta r$ during t_2, *i.e.*, i has been exposed during t_1+t_2 and $-(di/dr)\,\Delta r$ during t_2. This additional radiation has the same value with opposite signs at two opposite points of circle MM′. In the same way along circle NN′ intensity i has acted during t_1+t_2, and $(di/dr)\,\Delta r$ during t_2. Hence the radiation i was exposed during t_1+t_2 along a curve which lies between the two circles MM′ and NN′ and in such a way that the mean of the distances (ρ and ρ') from C of two opposite points of this curve is equal to r. Along

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this curve there is equal blackness on the photograph and the same blackness occurs on Photograph VI. at a point at distance h_6, which was illuminated by i_6 during t_6. Hence

$$i_6/i = \text{constant} = (t_1+t_2)^{1/a}/t_6^{1/a}.$$

The mean distance of the equal-blackness curve is the average value of ρ which is r. But r is, by assumption, the distance of the point on the corona at which the intensity is i. Hence the mean distance of the equal-blackness curve and h_6 are correlative distances on the corona. The same result holds good for the three eccentrically superposed pictures *abc* of the VIIth photograph, and the constant $F_{6,7}$ is equal to $(t_1+t_2+t_3)^{1/a}/t_6^{1/a}$. Photograph VII. may therefore be measured and reduced in the same way as the other photographs, provided that always two opposite points of an equal-blackness curve be measured. Terms of the second order have been neglected in this derivation; they amount to only a fraction of the distance AC (0·11 diameter) and are small quantities compared with the errors of measurement.

§ 7. *The Formula which gives the Intensity of the Corona as a Function of the Distance h.*

I first tried whether the observed distances satisfied Professor TURNER'S formula (intensity inversely proportional to the sixth power of the distances from the sun's centre), but find inadmissible residuals. Another formula has therefore to be derived. If the distances given in columns I. to V*a*., Table III., be plotted as ordinates, and the corresponding distances standing in the first column as abscissæ, the points belonging to the same column lie as nearly in a straight line as can be expected from the accuracy of the observations, and all these five lines can be made to intersect in a point $-x$, $-x$.

Hence

$$h_1+x = \gamma_n(h_n+x), \quad n = 1 \text{ to } 5,\ x \text{ a constant.}$$

The intensity i being a function of the distances h, which are counted from the sun's limb, I write $i = cf(h+x)$. Hence $i_1 = cf(h_1+x) = cf[\gamma_n(h_n+x)]$ and $i_n = cf(h_n+x)$,

$$\frac{i_1}{i_n} = \text{constant } F_{1,n} = \frac{f[\gamma_n(h_n+x)]}{f[h_n+x]},$$

as h_1 and h_n are correlative distances on the corona. This relation is satisfied by $f(z) = z^{-y}$. Hence $i = c(h+x)^{-y}$ and $F_{1,n} = \gamma_n^{-y}$. The formula is the same as Professor TURNER'S, with this difference, however, that x need not be the radius of the sun.

Approximate values of x and y are found in this way. I assume $x = 0$, 40, &c., to 320 (solar diameter = 1000) and calculate γ_n from h_1, h_n, and x. The residuals are

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$v = h_1+x-\gamma_n(h_n+x)$. I take the sum of the residuals irrespective of sign for $n = 2$ to 5. For each value of n the sum of the residuals is a minimum for x between 80 and 120, and the sum for all values of n together is a minimum for $x = 110$. The sum of the residuals is 50 per cent. greater for $x = 20$ and $x = 300$. $x = 500$ (radius of the sun) leaves inadmissible residuals.

The observed values of $F_{1,n}$ and γ_n belonging to $x = 110$ give $y = 3{\cdot}4$ for $n = 2$, $3{\cdot}3$ for $n = 3$, $3{\cdot}5$ for $n = 4$, and $3{\cdot}8$ for $n = 5$.

In the same way, if the distances obtained from Photograph No. VI*a*. be plotted as ordinates and those on Photograph V*b*. as abscissæ, a straight line represents the observations as well as any smooth curve that can be drawn, and the same remark refers to the distances obtained from Photographs VI*b*. and VII. The sum of the residuals is again small for values of x near 110, though the range of possible values of x is larger. The resulting value of y lies near 4.

I think this is sufficient proof that the function represents very nearly the observations. Let us assume it to be exactly correct. The method explains that x and the n constants γ are determined independently of the times of exposure from the condition $h_1+x/h_n+x = h'_1+x/h'_n+x = \gamma_n$, and that there are n equations for the unknown y. These n equations will be rigorously satisfied, provided the correct values of F be introduced, and hence $n-1$ values $F_{1,n}$ or $n-1$ values of the time of exposure can be determined from the equations along with y.

I prefer to determine x and y together by the Method of Least Squares. Let $x_0+\xi$, $y_0+\eta$ be the true values of x and y, and v_n be the accidental error of measurement of h_n, and ΔF_{mn} the correction of an approximate value F_{mn}, which need not necessarily be the calculated value [5 (*c*)]. The observations must rigorously satisfy the equations

$$\frac{h_n+v_n+x_0+\xi}{h_m+v_m+x_0+\xi} - (F_{mn}+\Delta F_{mn})^{1/y_0+\eta} = 0,$$

or

$$v_n - \frac{h_n+x_0}{h_m+x_0}v_m = -(h_n+x_0)+\xi+(F_{mn}+\Delta F_{m,n})^{1/y_0+\eta}(h_m+x_0).$$

The sum of the squares of the left side which contains the accidental errors is a minimum for the most probable values of ξ, η, and $(n-1)$ values ΔF_{mn}. The solution gives these unknowns as functions of one of the ΔF, or if all the ΔF be introduced as unknowns one of them must come out indeterminate. Instead of ΔF, I introduce the corrections of the adopted times of exposure Δt. For the first five photographs ξ, η, Δt_1, Δt_2, Δt_3 will be found as functions of Δt_4 and Δt_5, and for Photographs V*b*., VI., and VII., ξ, η, Δt_6 as functions of Δt_5 and Δt_7. Finally, all the time records can be used in determining the corrections Δt. It will be seen that the uncertainty of the exposure of Photograph VI. is not such a serious deficiency as might be expected at first sight.

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To determine by the Method of Least Squares only ξ and η would mean the discarding of the condition $(h_1+x)/(h_n+x) = (h'_1+x)/(h'_n+x)$ and must lead to erroneous results.

I calculate ξ, η, and Δt from equations which result from logarithmic differentiation of the equation given above. Let $t_n = t_n^0 + \Delta t_n$ ($n = 1$ to 6), $t_7 = 100{\cdot}88 - t_6^0 - \Delta t_6 + \Delta t_7$ (see § 6), $\alpha = 1{\cdot}05 + \Delta\alpha$, where t_n^0 ($n = 1$ to 5) are assumed to equal the observed values and t_6^0 is arbitrarily chosen equal to $11{\cdot}00$ (see § 6). The values F_{mn} are those appearing in § 5 (c), and they sufficiently approach their true values. I start from $x_0 = 140$, $y_0 = 4$, the result of a first solution. The equations of condition are

$$n = a\xi + b\eta + c_m\Delta t_m - c_n\Delta t_n + d\Delta\alpha,$$

where

$$n = \tfrac{1}{4}\log F_{mn} - \log\frac{h_n+140}{h_m+140}, \quad a = -\text{Mod}\,(1-F_{mn}^{-1/4})\,(h_m+140)^{-1},$$

$$b = \frac{1}{4^2}\log F_{mn}, \quad c_m = \text{Mod}\,(4\alpha t_m)^{-1}, \quad \text{and similarly for suffix } n,$$

but

$$c_6 = \text{Mod}\,(4\alpha)^{-1}\,(t_6^{-1} - t_7^{-1}) \quad \text{in equations } m = 6,\ n = 7,$$

$$d = (4\alpha)^{-1}\log F_{mn} \quad \text{for} \quad m = 5,\ n = 6 \quad \text{and} \quad m = 6,\ n = 7,$$

The weight p of an equation is calculated with r_m and r_n as given in Table III., 0·01 being the error of an equation of unit weight. The calculated weights served merely as a guide. The adopted weights appear in Table IV. The numerical equations are :—

Photographs.		
I.	II.	$n = -(8{\cdot}808)\,(h_1+140)^{-1}\,\xi + 0{\cdot}0174\eta + (9{\cdot}090)\,\Delta t_1 - (9{\cdot}111)\,\Delta t_2$
I.	III.	$n = -(9{\cdot}072)\,(h_1+140)^{-1}\,\xi + 0{\cdot}0344\eta + (9{\cdot}090)\,\Delta t_1 - (9{\cdot}122)\,\Delta t_3$
I.	IV.	$n = -(9{\cdot}239)\,(h_1+140)^{-1}\,\xi + 0{\cdot}0553\eta + (9{\cdot}090)\,\Delta t_1 - (9{\cdot}111)\,\Delta t_4$
I.	V*a*.	$n = -(9{\cdot}367)\,(h_1+140)^{-1}\,\xi + 0{\cdot}0834\eta + (9{\cdot}090)\,\Delta t_1 - (9{\cdot}085)\,\Delta t_5$
V*b*.	VI*a*.	$n = -(9{\cdot}297)\,(h_5+140)^{-1}\,\xi + 0{\cdot}0662\eta + (9{\cdot}085)\,\Delta t_5 - (7{\cdot}973)\,\Delta t_6 + (9{\cdot}401)\,\Delta\alpha$
VI*b*.	VII.	$n = -(9{\cdot}233)\,(h_6+140)^{-1}\,\xi + 0{\cdot}0543\eta + (8{\cdot}023)\,\Delta t_6 - (7{\cdot}061)\,\Delta t_7 + (9{\cdot}315)\,\Delta\alpha.$

n is entered in Table IV. The brackets indicate logarithms, -10 being omitted.

On account of the defects of Photograph VII. and the uncertainty of the exposure of VI., I have solved the equations appertaining to Photographs I. to V*a*. separately from those belonging to Photographs V*b*. to VII., and finally have discussed the whole material.

(a) *Photographs I. to Va.*—These determine the intensity curve from distance 60 to 520. In accordance with the above, two of the Δt are indeterminate. I choose Δt_4

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and Δt_5 and express the other unknowns as functions of them. The result of the solution is

$$
\begin{aligned}
x &= 140 \quad +9\cdot 2 \pm 16 \quad -22\cdot 2 \ \Delta t_4 + 21\cdot 2 \ \Delta t_5 \\
y &= 4\cdot 0 + 0\cdot 71 \pm 0\cdot 22 - 4\cdot 68 \ \Delta t_4 + 4\cdot 40 \ \Delta t_5 \\
t_1 &= 0\cdot 84 - 0\cdot 28 \pm 0\cdot 08 + 3\cdot 01 \ \Delta t_4 - 1\cdot 84 \ \Delta t_5 \\
t_2 &= 0\cdot 80 - 0\cdot 18 \pm 0\cdot 06 + 2\cdot 28 \ \Delta t_4 - 1\cdot 20 \ \Delta t_5 \\
t_3 &= 0\cdot 78 - 0\cdot 01 \pm 0\cdot 04 + 1\cdot 66 \ \Delta t_4 - 0\cdot 65 \ \Delta t_5 \\
t_4 &= 0\cdot 80 \qquad + 1 \ \Delta t_4 \\
t_5 &= 0\cdot 85 \qquad - 1 \ \Delta t_5
\end{aligned}
$$

The errors are mean errors. The mean error of an equation of unit weight is 0·014, as compared with the adopted value 0·010.

So far the time records have not been used (except in the calculations of the differential quotients, which is merely a matter of convenience). I determine Δt_4 and Δt_5 from all the time records, introducing the condition that the values of t differ from a mean value t_0 by accidental errors v. The equations are

$$
\begin{aligned}
t_0 - v &= 0\cdot 56 + 3\cdot 01 \, (\Delta t_4 - \Delta t_5) + 1\cdot 17 \, \Delta t_5 \\
t_0 - v &= 0\cdot 62 + 2\cdot 28 \qquad + 1\cdot 08 \\
t_0 - v &= 0\cdot 77 + 1\cdot 66 \qquad + 1\cdot 01 \\
t_0 - v &= 0\cdot 80 + 1\cdot 00 \qquad + 1\cdot 00 \\
t_0 - v &= 0\cdot 85 \qquad\qquad + 1\cdot 00
\end{aligned}
$$

The result is $\Delta t_4 - \Delta t_5 = +0{\cdot}096 - 0{\cdot}05 \, \Delta t_5$. The equations do not determine Δt_5 with any degree of accuracy. The unknowns then become

$$
\begin{aligned}
x &= 147 \quad \pm 16 \quad + 2 \ \Delta t_5 \\
y &= 4\cdot 26 \pm 0\cdot 22 - 0\cdot 05 \ \Delta t_5 \\
t_1 &= 0\cdot 85 \pm 0\cdot 08 + 0\cdot 98 \ \Delta t_5 \\
t_2 &= 0\cdot 84 \pm 0\cdot 06 + 1\cdot 07 \ \Delta t_5 \\
t_3 &= 0\cdot 93 \pm 0\cdot 04 + 0\cdot 93 \ \Delta t_5 \\
t_4 &= 0\cdot 90 \qquad + 0\cdot 95 \ \Delta t_5 \\
t_5 &= 0\cdot 85 \qquad + 1\cdot 00 \ \Delta t_5
\end{aligned}
$$

The value of Δt_5 is irrelevant for our purpose, it cannot be more than a fraction of a second and such a value changes x and y only by a small fraction of its error.

We may change y by a small quantity η of the order of its error and x by a corresponding quantity ξ without altering appreciably the residuals. The equations give $\xi = (1{\cdot}880)\,\eta$. I assume $y = 4$ and throw its error on x. The result is

(A) $$x = 127 \pm 23, \quad y = 4{\cdot}00.$$

(*b*) *Photographs Vb., VI. and VII.*—The photographs furnish material for the intensity-curve from $h = 110$ to about 1700 (1·7 solar diameter).

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Again two of the Δt remain indeterminate. I take $\Delta t_7 = 0$, which is permissible, as any reasonable error has a small effect on x and y, see (c), and express the unknowns as functions of Δt_5. The result is

$$x = 140 \quad -23 \quad \pm 46 \quad +1 \cdot 4 \ \Delta t_5$$
$$y = \quad 4 \cdot 00 - \ 0 \cdot 29 \pm \ 0 \cdot 32 - 1 \cdot 05 \ \Delta t_5$$
$$t_6 = \quad 11 \cdot 0 \ + \ 1 \cdot 37 \pm \ 0 \cdot 60 + 5 \cdot 45 \ \Delta t_5$$

The error of an equation of unit weight is 0·021.

The time records (except $t_6+t_7 = 100{\cdot}88$) have not yet been used. Δt_5 can be determined from the last equation.

The upper limit of t_6 [see § 6 (a)] is $11{\cdot}84-0{\cdot}1 = 11{\cdot}7$ and it gives $\Delta t_5 = -0{\cdot}12$ and the lower limit of t_6 is $11{\cdot}84-1{\cdot}0 = 10{\cdot}8$, which gives $\Delta t_5 = -0{\cdot}29$, both with an error of 0·11. The large value of Δt_5 belonging to the lower limit of t_6 is out of the question, because the pendulum of the contact clock could not possibly have been placed so much out of beat. Nevertheless, I maintain both values,

$t_6 = \ 11 \cdot 7$	$t_6 = \ 10 \cdot 8$
$x = 117 \quad \pm 46$	$x = 117 \quad \pm 46$
$y = \ 3 \cdot 84 \pm \ 0 \cdot 34$	$y = \ 4 \cdot 01 \pm \ 0 \cdot 34$
$t_5 = \ 0 \cdot 73$	$t_5 = \ 0 \cdot 56$

I again reduce x to $y = 4{\cdot}0$. The normal equation gives $\xi = (2{\cdot}107)\,\eta$, hence

(B)

$t_6 = \ 11 \cdot 7$	$t_6 = \ 10 \cdot 8$
$x = 137 \quad \pm 63$	$x = 116 \quad \pm 63$
$y = \ 4 \cdot 00$	$y = \ 4 \cdot 00$
$t_5 = \ 0 \cdot 73$	$t_5 = \ 0 \cdot 56$

The second result is not possible, as already mentioned.

(c) *All the Photographs I. to VII.*—In this solution I have included the unknown $\Delta\alpha$ in order to see what effect the error of α has on x and y. Five corrections ΔF can be found or five of the Δt, leaving two, say Δt_5 and Δt_7, besides $\Delta\alpha$, indeterminate. The result is :—

$$x = 140 \quad -2 \quad \pm 19 \quad -7 \quad \Delta t_5 + 0 \cdot 5 \quad \Delta t_7 - 20 \quad \Delta\alpha$$
$$y = \quad 4 \cdot 00 - 0 \cdot 15 \pm \ 0 \cdot 14 - 1 \cdot 1 \quad \Delta t_5 + 0 \cdot 009 \ \Delta t_7 - \ 4 \quad \Delta\alpha$$
$$t_1 = \quad 0 \cdot 84 + 0 \cdot 21 \pm \ 0 \cdot 08 + 1 \cdot 68 \ \Delta t_5 - 0 \cdot 006 \ \Delta t_7 + \ 2 \cdot 5 \quad \Delta\alpha$$
$$t_2 = \quad 0 \cdot 80 + 0 \cdot 19 \pm \ 0 \cdot 06 + 1 \cdot 46 \ \Delta t_5 - 0 \cdot 004 \ \Delta t_7 + \ 2 \cdot 0 \quad \Delta\alpha$$
$$t_3 = \quad 0 \cdot 78 + 0 \cdot 27 \pm \ 0 \cdot 05 + 1 \cdot 30 \ \Delta t_5 - 0 \cdot 003 \ \Delta t_7 + \ 1 \cdot 4 \quad \Delta\alpha$$
$$t_4 = \quad 0 \cdot 80 + 0 \cdot 17 \pm \ 0 \cdot 04 + 1 \cdot 16 \ \Delta t_5 - 0 \cdot 002 \ \Delta t_7 + \ 0 \cdot 8 \quad \Delta\alpha$$
$$t_5 = \quad 0 \cdot 85 \qquad\qquad +1 \quad \Delta t_5$$
$$t_6 = \quad 11 \cdot 00 + 1 \cdot 17 \pm \ 0 \cdot 37 + 5 \cdot 5 \quad \Delta t_5 - 0 \cdot 063 \ \Delta t_7 + \ 0 \cdot 25 \ \Delta\alpha$$
$$t_7 = 100 \cdot 88 - t_6 \qquad\qquad +1 \quad \Delta t_7$$

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The mean error of an equation of unit weight is found $= 0{\cdot}017$ as compared with the adopted value of $0{\cdot}010$. Any possible error Δt_7 cannot alter the value of the variables by more than a small fraction of their errors, and α was deduced from experiments with an error of $\pm 0{\cdot}01$. So far the time records have not been used; I determine Δt_5 as under (a) from the recorded times t_1 to t_5, neglecting Δt_7 and $\Delta\alpha$. The result is $\Delta t_5 = -0{\cdot}14 \pm 0{\cdot}06$. I substitute this value and calculate the errors on the supposition that $\Delta t_7 = \pm 1^s{\cdot}0$, $\Delta\alpha = \pm 0{\cdot}02$, which certainly exceed the true errors. The result is

$$x = 139 \pm 19, \quad y = 4{\cdot}00 \pm 0{\cdot}17,$$

t_6 becomes $11{\cdot}40 - 0{\cdot}50$, which lies between the limits derived for t_6 in § 6 (a). In this solution no use has been made of the time of exposure assigned to the sixth photograph.

The error of y may be combined with that of x (see above).

$$\text{(C)} \qquad x = 139 \pm 23, \quad y = 4{\cdot}00.$$

The results (A), (B), (C) agree very well; the good agreement of (A) and (B), which rest on different material, is remarkable. Considering that all the material contributed to (C), I might adopt it as final. I change x by a unit to round off the figure. Hence

$$\textbf{(D)} \qquad i = c\,(h + \mathbf{140} \pm \mathbf{23})^{-4}.$$

h is counted from the sun's limb in unit of 10^{-3} solar diameter and $\log c = 12{\cdot}228$ expresses the intensity in unit of the intensity of the corona at $h = 1000$. The residuals left by (D) and calculated with the corrected values of F appear in Table IV. under heading (v). I employ them to derive the errors of the distances. I divide the residuals in each column in three groups and regard the mean of the residuals in each group as the error of $\log(h_n + 140) - \log(h_m + 140)$, where h_n and h_m are the mean distances in each group. The errors of measurement will be about the same on the first two photographs and they can therefore be calculated. The calculations of the errors of h_n is sufficiently evident. The result is :—

h.	100.	200.	400.	600.	800.	1000.	1200.	1400.
Errors . .	1·4	5	15	27	47	70	100	130

These errors are almost twice as great as those in Table III. The excess must, I think, be mainly set down to systematic errors of measurement which are different for the several positives.

Fig. 7 shows a comparison of the intensity curve with the observations. As the observations do not give absolute intensities, but the ratio of the intensities at two correlative distances, I adopt at distances h_1, h_{5b}, and h_{6b} the intensities as calculated from the formula (D) and calculate the intensities at the correlative distances h_2, h_3, h_4, h_{5a}, h_{6a}, h_7, from the latter and the known ratios F.

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The differences between these intensities and the tabular intensities are the outstanding errors. The ratios of the intensities are calculated with 11·40 seconds for the sixth exposure and the recorded values of the times of exposure for the other

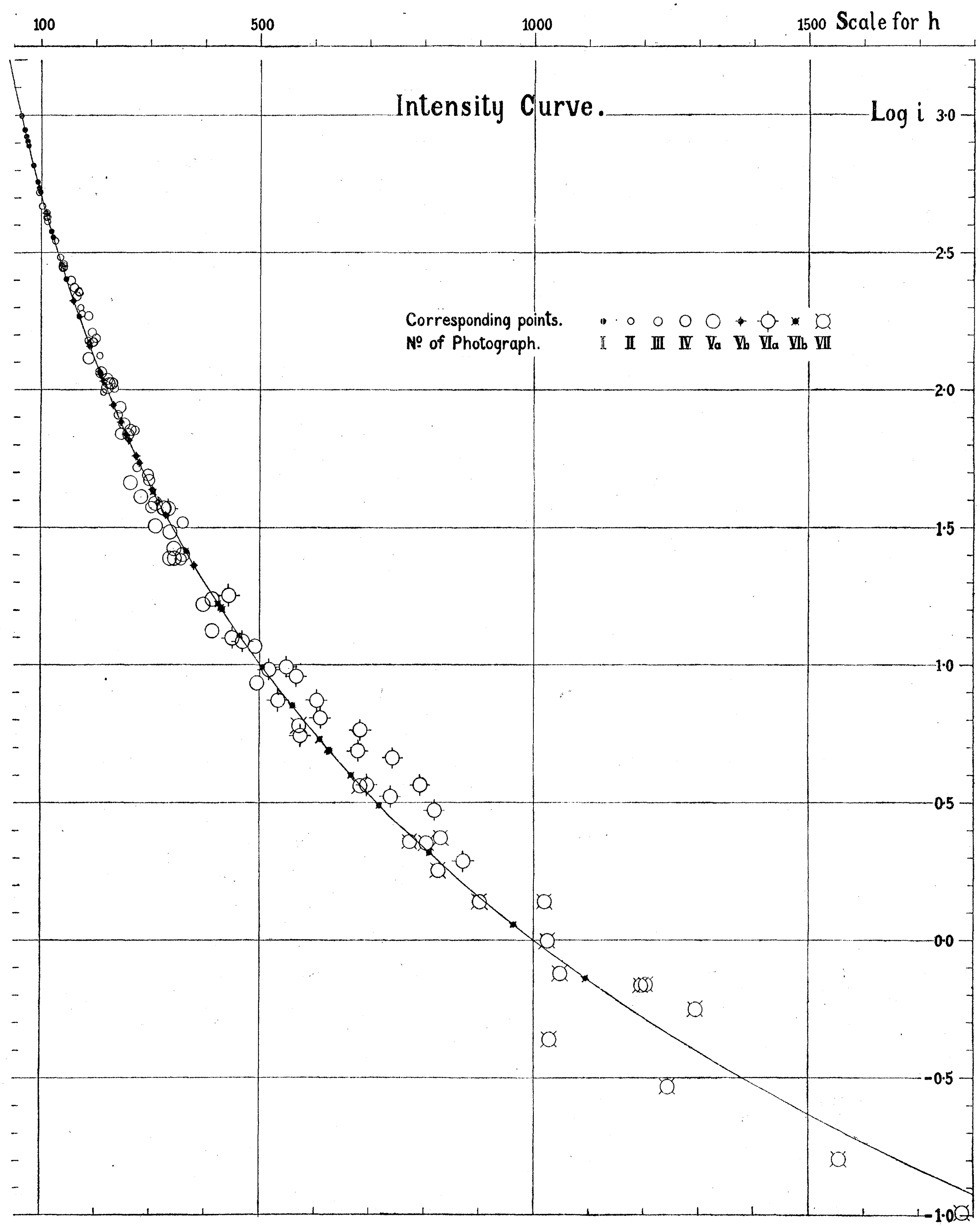

Fig. 7.

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exposures. The outstanding errors thus contain, apart from the errors of h, all the systematic errors arising from erroneous records of times of exposure. The points belonging to Photographs I., V*b*., VI*b*. which are placed on the curve are shown by dots, while the observed correlative points are marked by circles. The systematic errors are clearly reflected in these points.

§ 8. *Question whether or not the Formula* (D), § 7, *holds good at any Position-angle?*

Table I. gives at intervals of 15° of position-angle the amount δh by which the distance from the sun's limb of an equal-intensity curve exceeds the mean distance of that curve. Since these quantities were obtained from measurements on Photographs I., VIII., and IX., which have a uniform background all round, systematic errors of measurement will be eliminated in the differences, and their errors are more comparable to the errors given in Table III. than to those derived in last section. At a certain point of an equal-intensity curve, which is $h+\delta h$ from the sun's limb, the intensity is expressed by formula (D), in which h designates the mean distance of the intensity-curve. The intensity at the point may also be expressed by $(c+\delta c)(h+\delta h+140)^{-4}$, and if $c+\delta c$ be a constant, *i.e.* $\delta h/h+140 = \delta c/4c =$ constant at a series of points, the formula will hold good for these points. The values $a = 100\delta h/h+140$ are entered in Table II. The value of the constant $c+\delta c$, which gives at $h+\delta h$ a value of the intensity equal to that given by formula (D) involving c and the mean distance h, is given by $\delta \log c = 0{\cdot}017a$, $(0{\cdot}017 = 4 \text{ mod}/100)$. According to Table II. the quantities a differ for the same position-angle, and they vary systematically with the distance. I adopt for the same position-angle the same constant at all distances, and determine it by $\delta \log c = 0{\cdot}017a_0$, and hence the logarithm of the calculated intensity at $h+\delta h$ will differ by $(a_0-a)\,0{\cdot}017$ from that calculated by formula (D). I choose for a_0 the mean of the values a belonging to the same position-angle, and find that a_0-a lies between 0 and 4 for 90 per cent. of the number of points and, therefore, the difference of the intensities is from 0 to 17 ($\log 1{\cdot}17 = 0{\cdot}068$) per cent. of the intensity. An error of 17 per cent. in the intensity is equivalent to an error in distance h of 9 at $h = 100$, 21 at $h = 400$, 44 at $h = 1000$.

The errors of h belonging to formula $i = (c+\delta c)\ (h+\delta h+140)^{-4}$ are those thus derived combined with the errors given at the end of § 7. The residuals in h left by such an intensity curve would, therefore, be in excess of the errors of the observed values of h.

At some position-angles $\delta c/c$ changes little with the distance from the sun and therefore the formula represents the observations satisfactorily, and in some regions the representation would be improved if points lying on a curve be considered together. Whether these curves agree with the course of the streamers or not I have not investigated.

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§ 9. *On the Number of Particles and Intensity of Light per Unit Volume of the Corona.*

I shall explain that this problem can be solved on the following assumptions :—

(1) The luminosity of the corona is caused by particles, which are heated to incandescence by solar radiation, and which scatter sunlight.

(2) The number N (r) of particles per unit volume is a function of the distance r from the sun's centre.

(3) The apparent intensity is a known function of r [see formula (D), § 7].

(4) The ratio $g(r)$ of polarised and total light has been observed and represented as a function of r.

(5) The intensity of light, T (r), of a particle heated by solar radiation is correctly determined by STEFAN's and the Wien-Planck formulæ.*

With reference to (5) I have calculated the temperatures of particles at distances $h = 50, 100, 200, 300, 400, 600, \ldots, 1600$ from STEFAN'S formula (absolute temperature of the sun = 6000), and the intensities for wave-lengths 3000 to 5000. I find that their integral intensity T (r) appertaining to blue-violet light is very nearly inversely proportional to the sixth power of r, the average error of the intensities between $h = 50$ and 1200 being only 7 per cent. of the intensity.

I adopt the following notation :—

C = centre of sun, P = position of scattering particle, r = its distance PC (in unit of the sun's radius), $\theta = \frac{1}{2}\pi -$ angle CP Earth, $\mathrm{P}(r)\cos^2\theta$ = light polarised by a particle at P in direction θ, $\mathrm{S}(r) - \mathrm{P}(r)\cos^2\theta$ = total light scattered by a particle at P in direction θ, $\mathrm{F}(r) = \mathrm{N}(r)[\mathrm{T}(r)+\mathrm{S}(r)]$, $f(r) = \mathrm{N}(r)\,\mathrm{P}(r)$.

The functions are

$$(1) \qquad \begin{aligned} &\mathrm{S}(r) = c_2 2\left(\tfrac{4}{3} - r^{-1} - 3r^{-3}\right), \quad \mathrm{P}(r) = c_2\left(r^{-1} - r^{-3}\right),\dagger \\ &\mathrm{T}(r) = c_1 r^{-6}. \end{aligned}$$

Let us find by integration the total light emitted by a channel of unit section which runs in the direction towards the earth. I designate by $\rho\,(= r\cos\theta = h + 500/500)$ the shortest distance of this channel from C and introduce

$$g = \tfrac{360}{500}\rho^{-1} = \tfrac{360}{500} r^{-1}\sec\theta = 360\,(h+500)^{-1}.$$

The element of volume at P = $r\sec\theta\,d\theta = \frac{360}{500} g^{-1}\sec^2\theta\,d\theta$.

Unit volume at P sends light $\mathrm{F}(r) - f(r)\cos^2\theta = \mathrm{F}(g\cos\theta) - f(g\cos\theta)\cos^2\theta$.

The total light sent by all the particles in the channel towards the earth equals

$$(2)\quad \mathrm{C}(h+140)^{-4} = \mathrm{C}360^{-4}g^4(1-g)^{-4} = \tfrac{360}{500}g^{-1}2\left[\int_0^{\frac{\pi}{2}} \mathrm{F}(g\cos\theta)\sec^2\theta\,d\theta - \int_0^{\frac{\pi}{2}} f(g\cos\theta)\,d\theta\right].$$

* See ARRHENIUS, 'Lick Observatory Bulletin,' No. 58.

† See Dr. SCHUSTER, "On the Polarisation of the Solar Corona," 'M. N.,' vol. 40, p. 38 (6).

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The left side can be developed into a power series of g

(3) $$(1-g)^{-4} = \sum_0^\infty P_n g^n,$$

therefore F and f must also be power series of g:

(4) $$F(\rho) = F(g) = C' g^5 \Sigma Q_n g^n,$$

(5) $$f(\rho) = f(g) = C' g^5 \Sigma R_n g^n, \quad \text{where} \quad C' = \tfrac{500}{360^5 2} C.$$

Let

$$d_n = \int_0^{\frac{\pi}{2}} \cos^n \theta \, d\theta.$$

(6) $$d_{2a} = \frac{1\,.\,3\ldots 2a-1}{2\,.\,4\ldots\ 2a}\,\frac{\pi}{2}, \quad d_{2a+1} = \frac{2\,.\,4\ldots\ 2a}{1\,.\,3\ldots 2a-1}\,\frac{1}{2a+1}.$$

These substituted in (2) give

(7) $$\Sigma P_n g^n = \Sigma d_{n+3} Q_n g^n - \Sigma d_{n+5} R_n g^n$$

or

(I) $$\frac{P_n}{d_{n+3}} = Q_n - R_n + \frac{R_n}{n+5}.$$

The second integral in (2) gives the polarised light, while the left side equals the total light. Their ratio was designated by $q(\rho)$; hence

(II) $$q(\rho)(1-g)^{-4} = \Sigma d_{n+5} R_n g^n.$$

Provided $q(\rho)$ be observed as a function of ρ, *i.e.* of g, R_n can be calculated by (II), and Q by (I), *i.e.* $F(\rho)$ and $f(\rho)$ become known functions. Their values are, if $C'' = C'(360/500)^5$,

(III)
$$F(\rho) = N(\rho)[T(\rho) + S(\rho)] = C'' \rho^{-5} \Sigma Q_n \left(\frac{0{\cdot}72}{\rho}\right)^n,$$
$$f(\rho) = N(\rho)\,P(\rho) = C'' \rho^{-5} \Sigma R_n \left(\frac{0{\cdot}72}{\rho}\right)^n.$$

Substitute T, S, and P and find $N(\rho)$ and c_1/c_2. The problem can therefore be solved if $q(\rho)$ were known. I am unable to say whether the measurements of the polarised light made at the last eclipse suffice to determine this function.

With reference to (I), $2P_n/3d_{n+3}$ very nearly equal the coefficients of a binomial series, and it is not difficult to prove that

$$(1-g)^{-4{\cdot}528} > \tfrac{2}{3}\Sigma \frac{P_n}{d_{n+3}} g^n > (1-g)^{-4{\cdot}5}.$$

The exponent $y = 4$, of formula (D), is derived from the observations with an error of 0·3 (assuming $x = 140$ to be correct), hence the errors of the exponents in the

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above inequality are about ten times as much as the range of the exponents, and we may write

$$\Sigma \frac{P_n}{d_{n+3}} g^n = \tfrac{3}{2}(1-g)^{-4\cdot 5}.$$

(I), (4), and (5) give then, if $C''' = 1\cdot 5C''$,

$$F(\rho) - f(\rho) = C'''\rho^{-5}\left(1-\frac{0\cdot 72}{\rho}\right)^{-4\cdot 5} - \tfrac{1}{5}C'g^5\Sigma \frac{5}{n+5} R_n g^n,$$

which stands for the light radiated and scattered at right angles to the radial direction by the particles in unit volume at distance ρ from the sun's centre. Considering that the second term is only a fraction of the polarised light and the latter a fraction of the total light, $F(\rho) - f(\rho)$ nearly equals the first term. If there were no light scattered by the particles but only radiated, the number of particles per unit volume, $N(\rho)$, would, by (1), be proportional to $\rho(1-0\cdot 72\rho^{-1})^{-4\cdot 5}$. This result differs from that derived by ARRHENIUS, who based his calculations on $T(\rho) =$ constant.

10. *Plea for Repetition of such Observations as contained in this Paper and for Observations of the Light Polarised at Various Distances.*

(*a*) For wave-lengths 0·3 to 0·5 the radiation of a particle at $h = 50$ is 355 times as great as that at $h = 1000$, while for wave-lengths 0·55 to 0·65 this ratio is only 70. Blue-violet radiation is almost inversely proportional to the sixth power of the distance of the particle from the sun's centre (see § 9, 5), and for red-yellow radiation the power is only 4·3.

Hence if in addition to photographs on ordinary plates a series of photographs be taken with a colour screen on a plate sensitized for red-yellow rays another formula would be found which should lead (see last section) to the same number of particles per unit volume as that belonging to blue-violet radiation. Two such series of photographs, together with observations of the light polarised at various distances, would thus decide the debated question whether the luminosity is actually caused by minute particles which are heated to luminescence by solar radiation and which scatter sunlight.

(*b*) Though it is a fact that the brightness of the corona undergoes changes, we are ignorant whether the intensity of the corona at a certain distance in terms of that at unit distance is a constant or not. Inferences might be drawn from data such as contained in this paper and belonging to a series of eclipses which would advance our knowledge of the constitution of the corona and give us some idea of the causes which produce it. It is, of course, necessary that the plates have on all occasions the same relative sensitiveness in the different regions of the spectrum. (I employed Imperial special rapid plates.)

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I cannot finish this paper without expressing my indebtedness to the University Court of Glasgow for a grant of £100 towards the expenses of the expedition; to my companion, Mr. JOHN FRANKLIN ADAMS, who presented half of this sum to the Court and superintended the arrangements for the transport of the instruments; to Mr. ANDREW CROOKSTON, Glasgow, for his hospitality at his comfortable house at Kalaa and the help the employees of his firm extended to the expedition *en route;* to the Council of the Royal Dublin Society for the loan of a siderostat, and to my companion, Mr. HENRY A. MAVOR, M.Inst.C.E., Glasgow, who, in the capacity of physician, engineer, and adviser, took upon himself much of that work which is not mentioned, but is so important to the success of an expedition.

APPENDIX I.*

Diffraction due to the Screens.

For the first four exposures, each of about a second, the aperture of the lens is reduced by a perforated screen which has thirteen equal circular openings. The arrangement of these openings will be seen in fig. 1: there are six holes in the corners of a regular hexagon, one in the centre, and six others are equidistant from each two of them. The diffraction pattern of a star does not consist, as might be thought, of a series of detached images which lie on lines intersecting in a centre, but, as photographs of α Lyræ have proved, shows, apart from a central region, luminous rings at the same distances at which one opening produces them. On the photograph of α Lyræ rings are visible as far as 5π (linear value of $\pi = 9$) for the first screen, and on the eclipse photographs the prominences have certainly made no impression beyond 10π. Faint though the intensity of the rings be, it requires investigation whether a distant ring belonging to a point of the corona near the sun has an intensity comparable to that of a distant point on whose image the ring is superposed, or rather whether all the diffracted light together is not a negligible quantity. I shall show that it is small. The result would have been different if the exposures had been longer and more distant parts of the corona had been photographed with the screens.

Let P be a point in the focal plane and C the position of its central image. I introduce a rectangular system of co-ordinates XY in the plane of the screen, origin in centre of the central opening, and X-axis parallel to CP.

Let there be only two holes which lie diametrically opposite and whose centres have the x-co-ordinates $\pm x$, then the state of oscillation at P is given by

$$k\pi\rho^2 \frac{2}{u} J_1(u)\ 2\cos(rx)\sin\alpha,$$

* Postscript, added at the request of one of the Referees. The photographic experiments were subsequent to and confirmatory of the mathematical analysis.

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where ρ designates the radius of the opening, θ the angular distance of CP at the centre of the object-glass, λ the wave-length, $r = \frac{2\pi}{\lambda}\sin\theta$, $u = r\rho$, and $J_1(u)$ BESSEL'S function of order 1. Let there be thirteen holes arranged as defined above and the distance between each two be equal to a, and let a diagonal of the hexagon and the x-axis enclose angle ϕ, then the state of oscillation at P is given by

$$\sin\alpha\left\{k\pi\rho^2\frac{2}{u}J_1(u)\left[1+2\cos(ra\cos\phi)+2\cos\left(ra\cos\left(\frac{\pi}{3}+\phi\right)\right)+2\cos\left(ra\cos\left(\frac{\pi}{3}-\phi\right)\right)\right.\right.$$
$$\left.\left.+2\cos(ra\sqrt{3}\sin\phi)+2\cos\left(ra\sqrt{3}\cos\left(\frac{\pi}{6}+\phi\right)\right)+2\cos\left(ra\sqrt{3}\cos\left(\frac{\pi}{6}-\phi\right)\right)\right]\right\}.$$

The intensity at P is the square of the coefficient of $\sin\alpha$. The position of point P is determined with reference to C by its linear distance $\zeta = f\sin\theta$ (f = focal-length) and its position-angle ϕ counted from a line parallel to one of the diagonals of the hexagon of the screen. For same values of ζ the intensity is the same for $\pm\phi$ and it is periodical with reference to ϕ, with a period of $\pi/3$. Hence the intensity can be developed into a cosine-series progressing by multiples of 6ϕ.

To find the quantity Q of light falling on a ring round C limited by radii ζ_1 and ζ_2, I multiply the intensity by the element $\zeta\, d\zeta\, d\phi$ of the area in the focal plane and integrate from $\phi = 0$ to 2π and from ζ_1 to ζ_2. The integration with reference to ϕ can be carried out. The result is, if u be introduced instead of ζ,

$$\zeta = \frac{f\lambda}{2\rho}\frac{u}{\pi}, \qquad u' = u\frac{a}{\rho},$$

$$Q_{u_1u_2} = k^2(f\lambda)^2(13\pi\rho^2)\,2\int_{u_1}^{u_2}\frac{(J_1(u))^2}{u}\,du$$
$$+k^2(f\lambda)^2(\pi\rho^2)\,12\int_{u_1}^{u_2}\frac{(J_1(u))^2}{u}\,du\,[8J_0(u')+5J_0(2u')+2J_0(3u')$$
$$+6J_0(\sqrt{3}u')+J_0(2\sqrt{3}u')+4J_0(\sqrt{7}u')].$$

J_0 designates BESSEL'S function of order zero. I transform the second integral. The values of J_0 and J_1 are with sufficient accuracy for values of u larger than π,

$$J_0(x) = \sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{x}}\sin\left(x+\frac{\pi}{4}\right), \qquad (J_1(x))^2 = \frac{1}{\pi}\frac{1}{x}(1-\sin 2x\ldots).$$

The terms in [ ] have, for the first screen, respectively the periods 70°, 35°, 23°, 40°, 20°, 26°, and owing to these short periodic terms the quantity to be integrated changes sign at small intervals of u. To a given value of u_1, say, $= n\pi$, a limit u_2 near $(n+1)\pi$ can be found which makes the second integral zero. I have convinced myself by mechanical quadrature that this deduction is correct even for $u_1 = 0$, $u_2 = \pi$, ..., $u_1 = 3\pi$, $u_2 = 4\pi$. For our purpose it is unnecessary to take the second part into account. Therefore, if all the light falling on the ring be considered

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together the distribution is almost exactly the same as if all the light had passed through only one of the openings; and in accordance with the above, if the ring be divided into 12 parts by 6 diameters, beginning at $\phi = 0$, each segment contains a twelfth of the light falling on the whole ring.

The quantity of light falling on such a twelfth of a zone between $u = n\pi$ and $(n+1)\pi$ is given by

$$Q_n = Ci\pi\rho^2 \tfrac{1}{12} f_n, \quad \text{where} \quad f_n = 2\int_{n\pi}^{(n+1)\pi} \frac{[J_1(u)]^2}{u} du.$$

i designates the intensity outside the object-glass.

I next consider the light Q falling on unit area at a point C of the image of a luminous area. Draw circles, radii $n\pi$, round C, and divide each ring by 6 diameters into 12 equal parts. Project from centre of object-glass this system of circles and lines on the luminous area. Let the intensity $i_n{}^m$ of the source be constant within an area of the source corresponding to part m of ring n; then, if there were x units in one of these parts, each unit would send Q_n (for $i = i_n{}^m$) divided by x to unit at C, *i.e.* the x units send Q_n to unit at C. The total quantity of light falling on unit at C is therefore given by

$$Q = C\pi\rho^2 \left[f_0 i_0 + \sum_{n=1}^{\infty} \frac{f_n}{12} \left(i_n{}^{(1)} + i_n{}^{(2)} + \ldots + i_n{}^{(12)} \right) \right]$$

or

$$Q = C\pi\rho^2 (i_0 + \Delta i),$$

where

$$\Delta i = \sum_1^{\infty} \frac{f_n}{12} \left(i_n{}^{(1)} + i_n{}^{(7)} - 2i_0 \right) + \ldots \frac{f_n}{12} \left(i_n{}^{(6)} + i_n{}^{(12)} - 2i_0 \right),$$

because $\sum_0^{\infty} f_n = 1$. Parts 1 and 7, 2 and 8, &c., lie diametrically opposite with reference to C. Hence the quantity of light at C is not changed by diffraction if the source be everywhere equally intense, or if the intensity uniformly increase along the lines drawn through C. In the case of the corona the second condition is very nearly satisfied in the neighbourhood of a point, and thus the most luminous diffraction rings hardly affect the quantity of light at C, there being almost as much light lost as gained.

Let Δi be known for each of the four screens (*i.e.* ρ), and at each distance h. Equal blackness was observed on two photographs exposed equally long with screens a and b, at two points h_a and h_b; hence $Q_a = Q_b$, and

$$\frac{C\pi\rho_a{}^2}{C\pi\rho_b{}^2} \frac{i_a + \Delta i_a}{i_b + \Delta i_b} = 1,$$

or

$$\frac{i_a + \Delta i_a}{i_b + \Delta i_b} = F_{a,b} \quad [F_{a,b}, \text{ see } \S\ 5\ (c)].$$

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The distances $h+\Delta h$ at which the corona has the intensities $i+\Delta i$ are correlative distances on corona [compare § 5 (d) and (f)], where $\Delta h = +\Delta i \dfrac{dh}{di}$.

The intensity formula for the corona ought to have been derived from the observed values of h corrected by Δh.

As to the calculation of Δh, I obtained the intensities of the corona from a diagram. I drew six lines through C at intervals of 30° and marked off points at distances $(n+\frac{1}{2})\,\pi$ from C. I assume that the intensity of the corona belonging to a point thus marked equals the mean intensity at all the points lying within a ring limited by circles $n\pi$ and $(n+1)\,\pi$ and up to 15° from it. The intensity at the points was read off the diagram and multiplied by $f_n/12$. The linear value of π is, in unit of 10^{-3} solar diameter, 9·1 for screen 1, 6·5 for screen 2, 4·7 for screen 3, 3·2 for screen 4, 0·55 for full aperture. For screens 3 and 4 several rings were treated together. In some directions the calculation had to extend as far as ring 80π. I calculated f by the following formulæ

$$2\int_0^{n\pi}\frac{[J_1(u)]^2}{u}\,du = 1-[J_0(n\pi)]^2-[J_1(n\pi)]^2 = 1-r_n,$$

$$J_0(n\pi) = \frac{1}{\sqrt{n\pi}}\left(1-\frac{1}{8n\pi}\right),\quad J_1(n\pi) = \frac{1}{\sqrt{n\pi}}\left(1-\frac{3}{4n\pi}\right) \text{ for large } n,$$

$$f_n = r_{n+1}-r_n.$$

For small values of n I interpolated the value of the integral from the table given in MUELLER's 'Photometrie der Gestirne,' p. 166.

The result of the calculation is—

Screen . .	1			2		3		4	
h	65	110	160	110	210	160	270	210	350
Δh	+0·7	−0·4	−0·9	−0·3	−1·1	−0·5	−1·0	−0·5	−1·5
n	1·2	1·7	3·5	1·7	5	3·5	8	5	12

The systematic errors Δh due to diffraction, and still more their functions v [see § 5 (f)], are so small compared with the accidental errors n of measurement, as calculated from the residuals, that they can be neglected, and hence formula (D), § 7, gives the relative intensities of the corona.

APPENDIX II.

Comparison of Corona and Moon.

The results contained in this section are not to be considered as an attempt to standardise my eclipse plates, but they originated in a desire to give future observers some ideas of the intensities with which they have to deal.

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After my return to the Observatory I photographed the moon on three nights with the eclipse apparatus and approximately at the same zenith distance the sun had at the eclipse. The atmosphere was exceptionally transparent for Glasgow on the first and third nights. I used plates returned from Kalaa and I developed them in the same way and at the same temperature as the eclipse photographs.

(*a*) *Brightness of Corona.*—The plates show, just as the eclipse photographs, a background due to diffused light. I compared the intensity of the background of the lunar photographs with those on the eclipse negatives, picking out those exposures which showed the same density of background in both. The durations of exposure give then the ratio of the light of the sky when illuminated by the corona and that when illuminated by the moon, and this equals the ratio of the total light emitted by the corona and the moon, provided the diffused light at the eclipse is exclusively due to the corona and the relative intensities of the two spectra are the same. Assuming ZÖLLNER's observations of the luminosity of lunar phases in terms of that of full moon, I find from 17 comparisons that the total light of the corona equals seven full moons. The comparison belongs to blue-violet rays.

(*b*) *The Intensity of the Corona in Terms of Lunar Intensity.*—Let i_a be the intensity of the region of the moon which lies north and south of Grimaldi and close to the edge of the moon. I compared the blackness of this region on the photograph with that of the corona on that photograph which was equally long exposed and through the same aperture, and measured the distances of points of the corona at which both showed the same degree of blackness. With the reduced distances I calculate, by formula (D), i/c, which equals i_a/c. Instead of i_a, which belongs to phase angle α, I introduce i_g, the intensity of the Grimaldi region at mean full moon, and find $\log i_g/c$ equal to 2·543 from nine photographs on October 18, 2·578 from three photographs on November 14, and 2·532 from 17 photographs on November 15. The mean 2·551 belongs to $h = 122$, and at this distance from the sun's limb the intensity of the corona equals that of the Grimaldi region at mean full moon. Therefore the constant of formula (D) is $\log c = 12{\cdot}228 - 2{\cdot}551 + \log i_g = 9{\cdot}677 + \log i_g$. For want of suitable apparatus I am unable to measure i_g in terms of the average intensity of full moon, but I am led to expect by integration of the intensity formula and comparison with the total light of the corona (seven full moons) that i_g is about 4. $i_g = 4$ would make the intensity of the corona at a distance of 0·23 diameter equal to that of full moon, a result which is quite at variance with that cited by LANGLEY.*

* 'The 1900 Solar Eclipse Expedition.'

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TABLE I.—[See § 4 (*c*).] Equal-intensity Curves of the Corona.

Mean distance	50	200	400	600	800	1000
Position-angle from N. of sun.	Observed minus mean distances. Unit = 0·001 solar diameter.					
0	0	+ 6	+46	+78	+53	− 7
15	+ 4	+15	+43	+42	+62	+ 13
30	+ 6	+13	+13	+37	+26	+ 4
45	+ 7	+11	+39	+43	+44	+ 59
60	+ 9	+24	+40	+48	+39	+ 19
75	+ 4	+37	+27	+49	+15	− 54
90	+21	+11	− 2	− 9	+ 4	+ 9
105	+11	+11	−11	−22	−12	+ 11
120	+ 4	+ 6	−11	−28	−13	+ 6
135	+ 4	+ 4	0	−18	−11	+ 11
150	− 4	−11	− 5	− 9	−20	− 57
165	−24	−39	−50	−46	−50	− 37
180	−18	−24	−28	−38	−26	− 70
195	−11	−26	−32	−54	−34	− 82
210	−17	−48	−76	−74	−74	− 61
225	− 4	−37	−68	−74	−94	−120
240	− 4	−18	−50	−57	−67	+ 11
255	0	− 8	−41	−54	−45	− 28
270	0	+13	−15	−28	+ 8	+ 35
285	0	+15	+11	+ 1	+24	+ 96
300	+ 6	+22	+57	+60	+63	+159
315	+ 4	+20	+50	+59	+52	+ 54
330	0	− 4	+11	+33	+28	+ 55
345	+ 2	+ 7	+52	+61	+28	− 26

TABLE II.—(See § 8.) $\frac{100\,\delta h}{h+140}$.

Mean distance h	50	200	400	600	800	1000
Position-angle.						
0	0	+ 2	+ 9	+11	+6	− 1
30	+ 3	+ 4	+ 2	+ 5	+3	0
60	+ 5	+ 7	+ 8	+ 7	+4	+ 2
90	+11	+ 3	0	− 1	0	+ 1
120	+ 2	+ 2	− 2	− 4	−1	+ 1
150	− 2	− 3	− 1	− 1	−2	− 5
180	− 9	− 7	− 5	− 5	−3	− 6
210	− 9	−14	−14	−10	−8	− 5
240	− 2	− 5	− 9	− 8	−7	+ 1
270	0	+ 4	− 3	− 4	+1	+ 3
300	+ 3	+ 7	+11	+ 8	+7	+14
330	0	− 1	+ 2	+ 5	+3	+ 5
90 to 270	− 2	− 4	− 6	− 6	−4	− 3
270 „ 90	+ 2	+ 4	+ 6	+ 6	+4	+ 3

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TABLE III.—[See § 4 (*d*).] Mean Corresponding Distances (*h*) from the Sun's Limb of Points of the Corona at which the Photographs show Equal Blackness. (Unit = 0·001 Solar Diameter.)

Photographs I. to V*a*.														
I.			II.			III.			IV.			V*a*.		
h.	*r*.	*p*.	*h*.	*r*.	*p*.	*h*.	*r*.	*p*.	*h*.	*r*.	*p*.	*h*.	*r*.	*p*.
63	0·2	24	96	0·2	24	139	2·0	24	187	2·4	24	263	4	10
69	1·1	24	102	1·6	24	154	1·3	24	209	1·9	24	283	3	11
72	1·5	24	111	1·1	24	161	1·7	24	220	1·7	21	309	4	11
74	1·5	24	109	1·8	24	170	2·4	24	224	2·8	21	324	6	11
74	1·5	24	111	2·4	24	169	1·7	24	228	2·8	21	326	7	11
76	1·1	24	111	1·5	24	165	1·3	24	220	1·3	21	309	4	11
85	2·2	24	126	2·2	24	187	3·3	24	244	3·5	21	335	6	8
93	1·8	24	135	1·7	24	193	2·6	24	252	2·6	21	343	6	8
96	1·7	24	141	3·0	24	200	2·6	24	263	3·7	21	359	11	8
98	1·5	24	139	1·7	24	194	1·9	24	246	2·6	21	335	11	8
98	1·8	24	141	1·7	24	196	2·0	24	259	3·0	21	344	4	8
119	2·4	24	172	2·2	24	232	4·3	21	296	5·0	21	413	6	8
122	1·7	24	174	1·7	24	233	1·7	21	298	2·8	21	396	9	8
137	2·8	24	185	2·6	24	241	3·7	21	302	5	21	413	9	8
146	2·4	24	207	2·6	24	272	3·0	21	359	5	19	491	10	6
169	3·0	24	213	3·0	24	276	4·3	21	356	6	19	494	17	5

Photographs V*b*. and VI*a*.						Photographs VI*b*. and VII.					
V*b*.			VI*a*.			VI*b*.			VII.		
h.	*r*.	*p*.	*h*.	*r*.	*p*.	*h*.	*r*.	*p*.	*h*.	*r*.	*p*.
*109	2	15	*333	10	11	*306	7	7	*572	6	10
159	5	15	443	5	10	365	8	7	683	17	9
187	5	11	450	4	10	424	11	4	831	18	9
*189	3	12	*469	5	11	428	7	4	774	17	8
207	4	9	548	23	10	*430	10	5	*804	11	7
209	4	8	517	8	9	463	20	4	826	19	3
214	6	9	567	5	7	504	13	4	902	15	3
232	4	7	533	14	7	*504	11	4	*1020	24	5
232	4	8	604	13	7	559	10	4	1026	44	4
*246	5	10	*611	7	9	609	35	4	1048	15	3
256	5	10	683	24	7	626	20	4	1204	14	4
261	3	6	574	9	7	*628	37	4	*1196	30	4
274	11	7	680	19	7	*667	1	2	*1295	15	3
*280	4	8	*743	15	8	719	17	3	1028	35	3
304	3	4	696	9	6	809	28	4	1244	111	3
304	12	8	793	30	5	*963	26	2	*1556	48	3
315	9	7	739	9	7	*1094	—	1	*1778	60	3
328	19	5	819	20	5						
380	20	3	872	33	5						

* Measured on enlargements.

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TABLE IV.—(See § 7.) Weights (p) and Residuals (n, v) of Equations.

I. and II.			I. and III.			I. and IV.			I. and Va.			Vb. and VIa.			VIb. and VII.		
p.	$n10^3$.	$v10^3$.	p.	$n10^3$.	$v10^3$.	p.	$n10^3$.	$v10^3$.	p.	$n10^3$.	$v10^3$.	p.	$n10^3$.	$v10^3$.	p.	$n10^3$.	$v10^3$.
2	+ 4	+ 7	2	− 1	+15	2	+14	+19	2	+36	+21	1	−14	+ 8	1	+14	+10
2	+ 6	+ 8	2	−11	+ 4	2	− 2	+ 3	2	+28	+13	1	−25	− 3	1	+ 5	+ 1
2	− 4	− 1	2	−15	0	2	− 9	− 4	2	+ 8	− 7	1	+ 9	+31	1	−19	−23
2	+ 4	+ 6	2	−23	− 8	2	−10	− 5	1·5	− 2	−17	1·5	− 3	+20	1	+11	+ 6
2	0	+ 3	2	−22	− 7	2	−14	− 9	1·5	− 4	−19	1·5	−33	−10	1	− 2	− 6
2	+ 4	+ 7	2	−12	+ 3	2	− 1	+ 4	2	+16	+ 1	1·5	−10	+12	0·5	+12	+ 8
2	− 3	− 1	2	−25	−10	2	−11	− 6	1·5	+ 9	− 6	1	−36	−13	1	+ 8	+ 4
2	− 2	0	2	−17	− 2	2	− 5	0	1·5	+17	+ 2	1	+ 7	+30	0·5	−38	−43
2	− 6	− 4	2	−21	− 6	2	−11	− 6	1	+ 8	− 6	1	−36	−14	0·5	− 5	− 9
2	0	+ 3	2	−10	+ 6	2	+11	+16	1	+34	+19	1·5	−24	− 2	0·2	+17	+13
2	− 3	0	2	−12	+ 3	2	− 3	+ 2	1·5	+25	+11	0·5	−53	−31	0·5	−27	−31
2	−11	− 9	2	−20	− 4	1·5	− 5	0	1·5	+ 4	−11	1	+14	+36	0·2	−23	−27
2	− 9	− 7	2	−16	− 1	2	− 2	+ 3	1	+23	+ 8	0·5	−34	−12	0·5	−33	−37
2	0	+ 3	1·5	− 1	+14	1·5	+18	+23	1	+33	+19	1	−58	−36	0·5	+84	+80
2	−14	−12	2	−21	− 6	1·5	−21	−16	1	−10	−25	1·5	−10	+12	0·2	+53	+49
2	+12	+14	1·5	+ 8	+24	1·5	+16	+21	1	+22	+ 7	0·5	−58	−36	0·5	+30	+26
												1	−21	+ 1	0·5	+26	+21
												0·5	−47	−25			
												0·5	−24	− 2			

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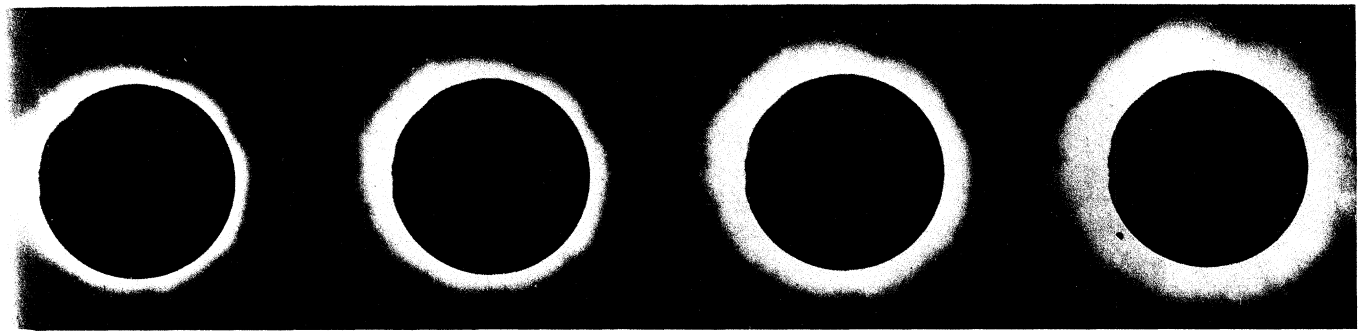
NINE AUTOMATIC EXPOSURES ON ONE PLATE.



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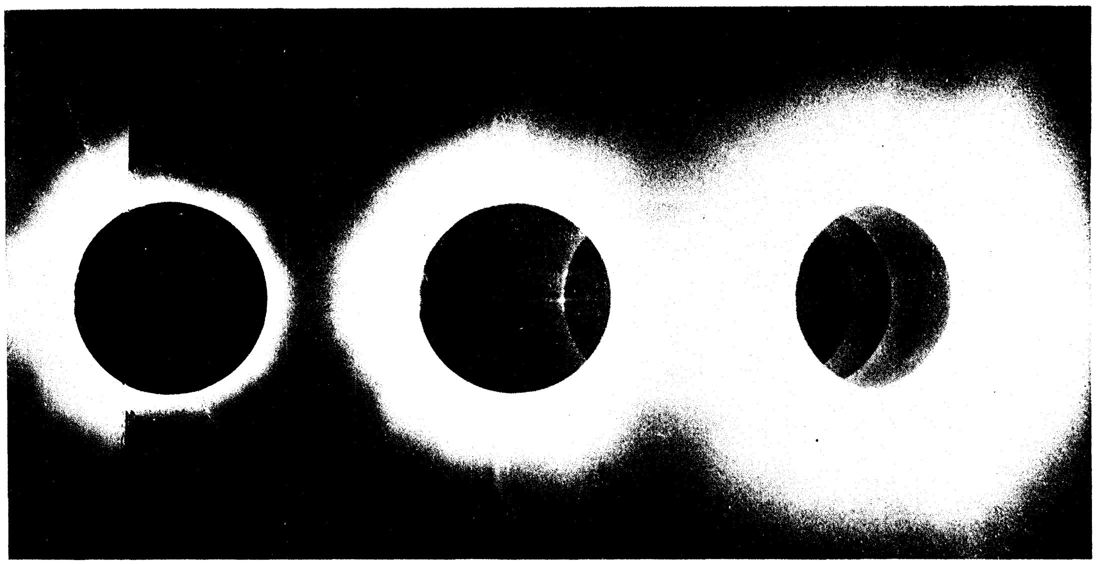

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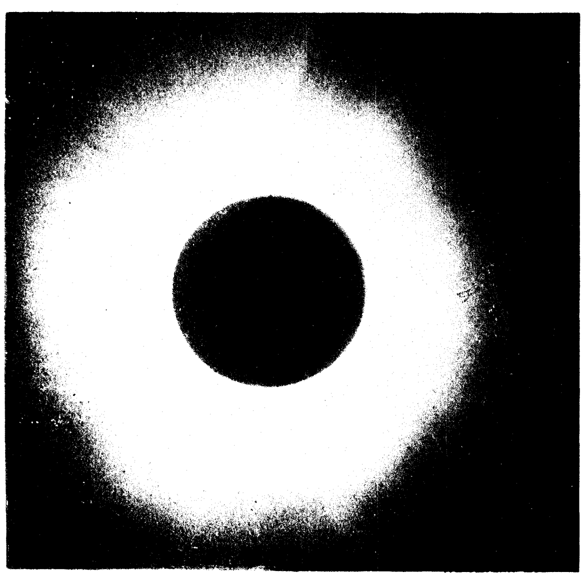

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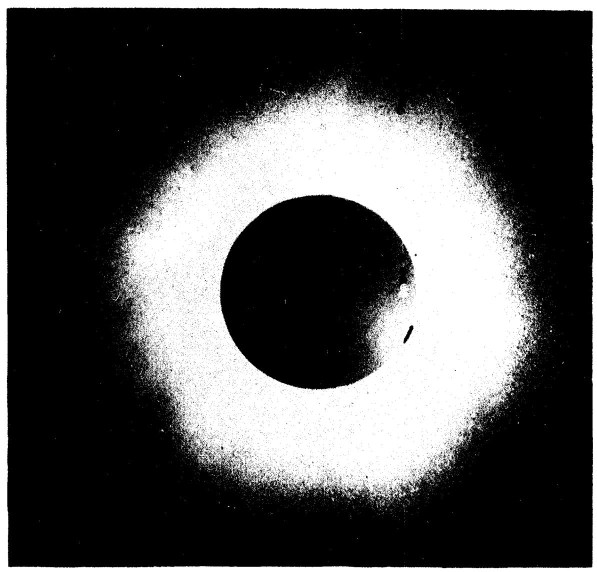

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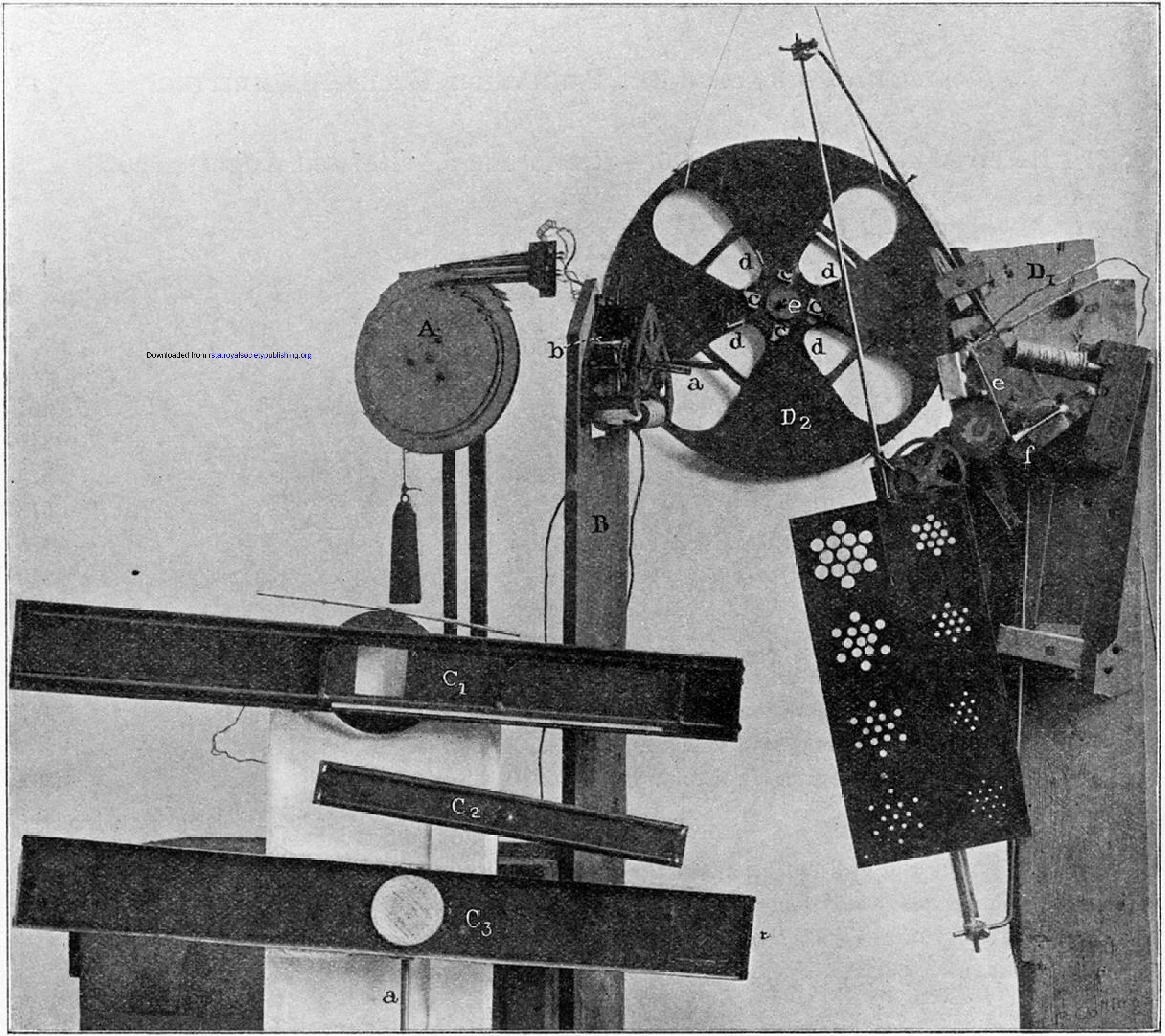

Fig. 1.

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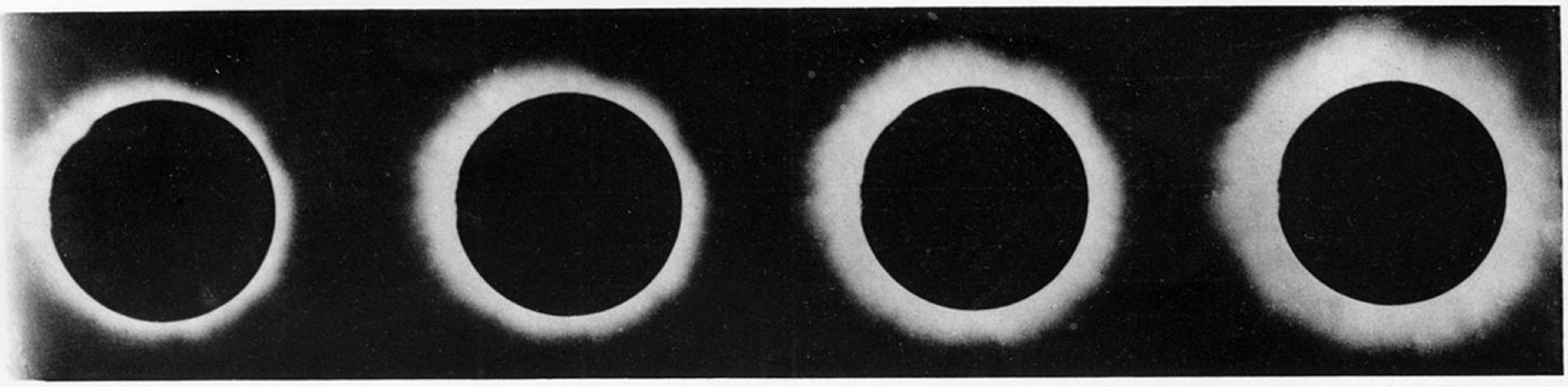

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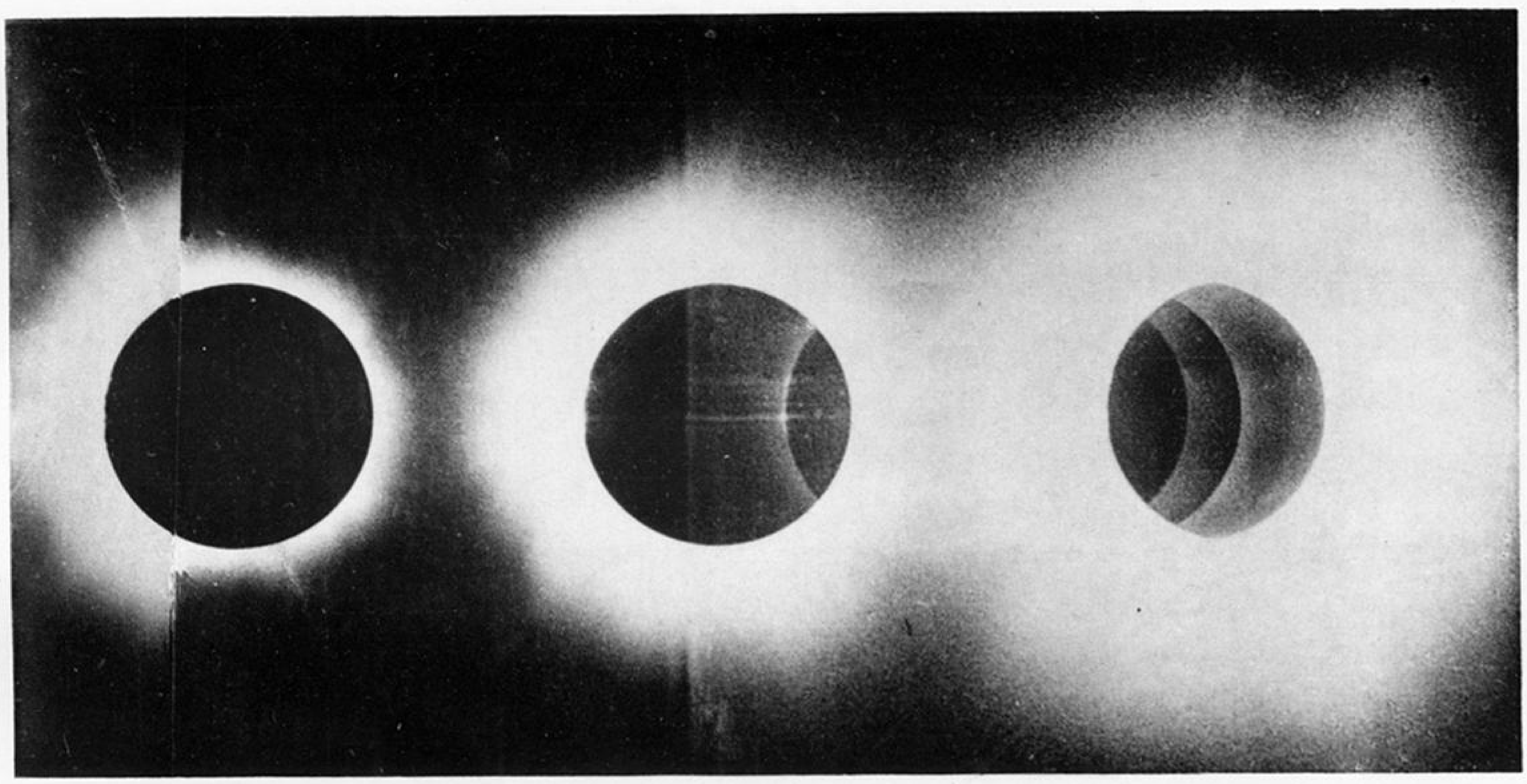

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16·7	16·7	16·7

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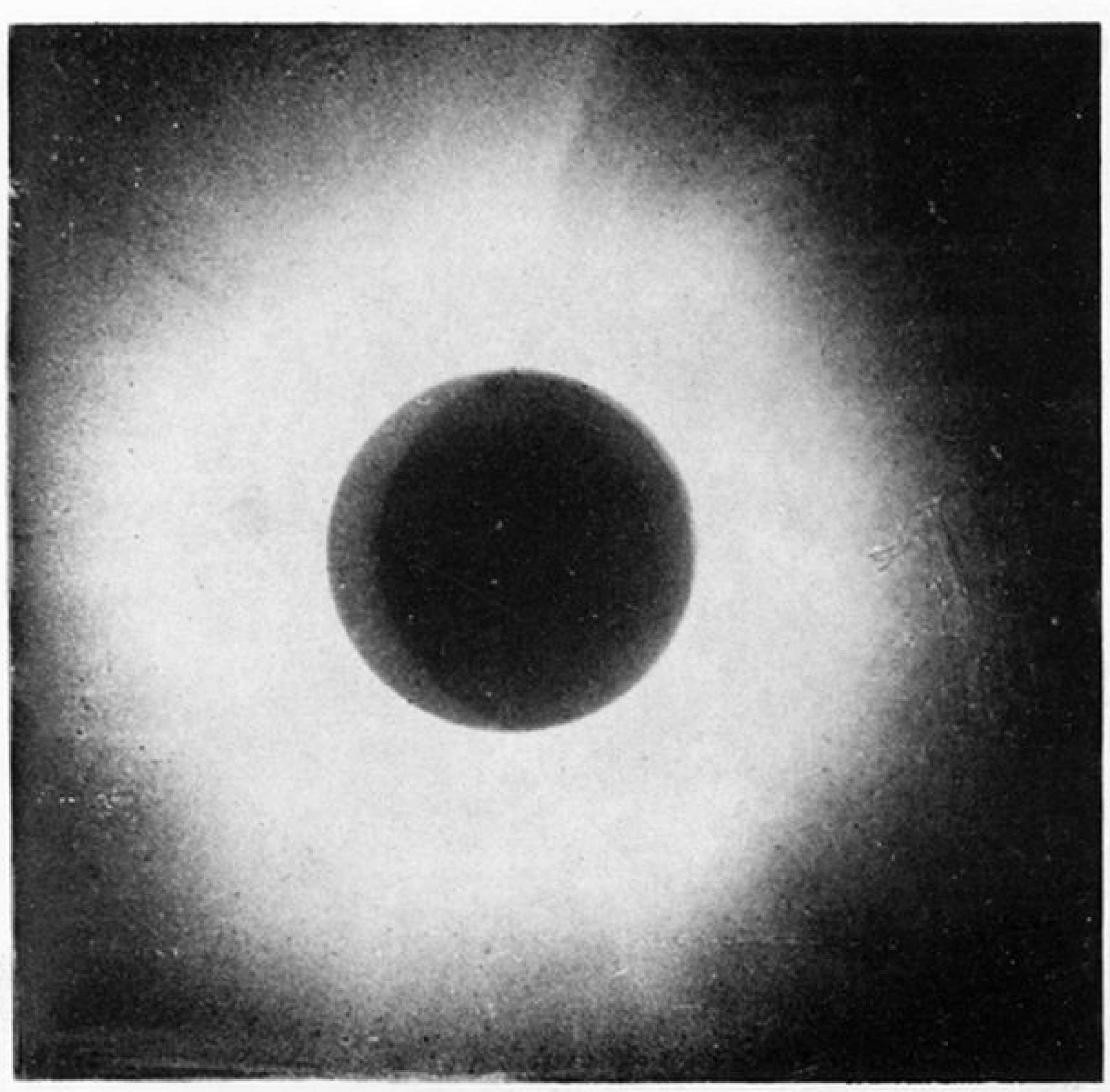

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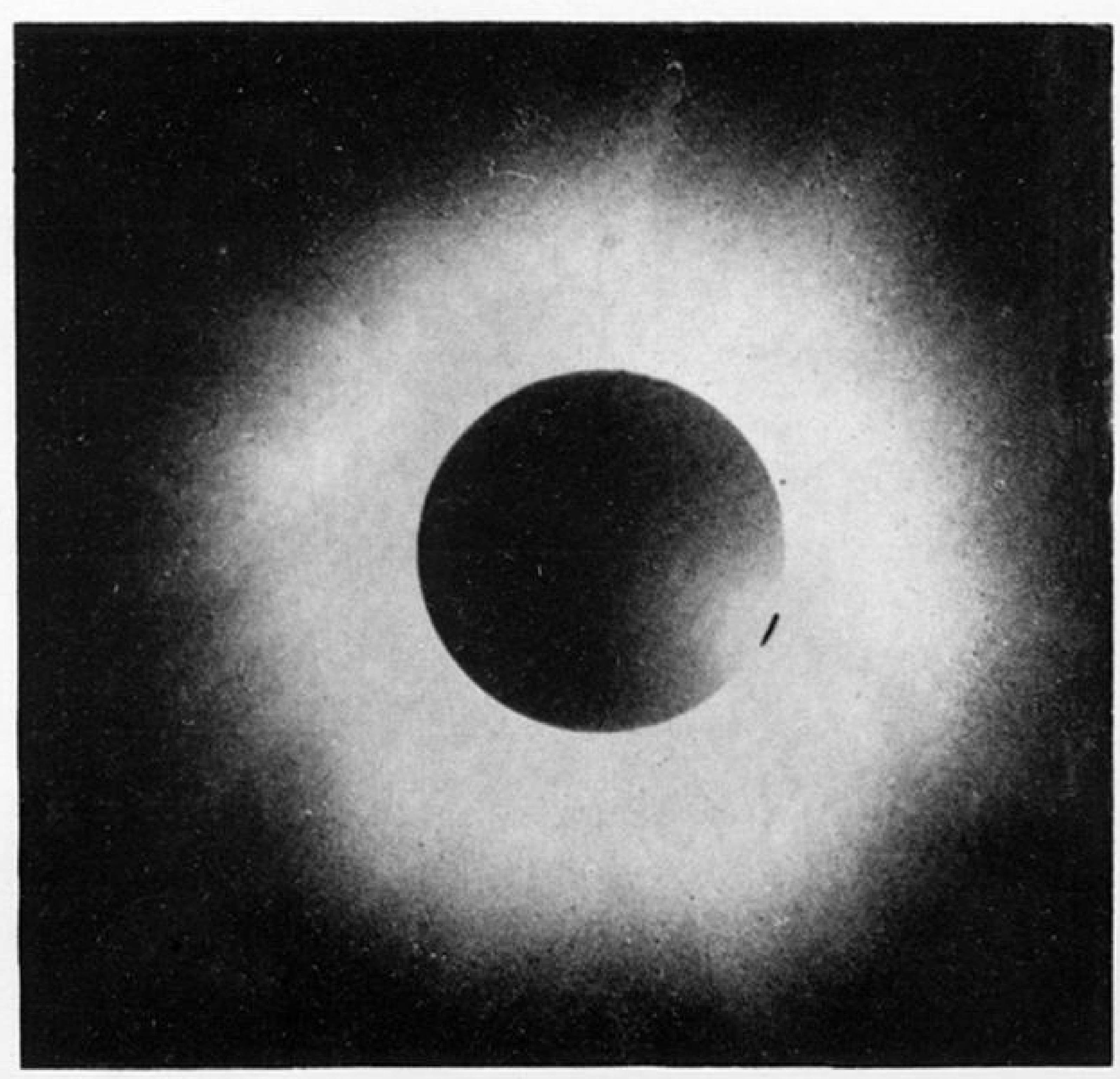

43 sec.
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